

CHAPTER

Free Convection

9



In preceding chapters we considered convection transfer in fluid flows that originate from an *external forcing* condition. For example, fluid motion may be induced by a fan or a pump, or it may result from propulsion of a solid through the fluid. In the presence of a temperature gradient, *forced convection* heat transfer will occur.

Now we consider situations for which there is no *forced* velocity, yet convection currents exist within the fluid. Such situations are referred to as *free* or *natural convection*, and they originate when a *body force* acts on a fluid in which there are *density gradients*. The net effect is a *buoyancy force*, which induces free convection currents. In the most common case, the density gradient is due to a temperature gradient, and the body force is due to the gravitational field.

Since free convection flow velocities are generally much smaller than those associated with forced convection, the corresponding convection transfer rates are also smaller. It is perhaps tempting to therefore attach less significance to free convection processes. This temptation should be resisted. In many systems involving multimode heat transfer effects, free convection provides the largest resistance to heat transfer and therefore plays an important role in the design or performance of the system. Moreover, when it is desirable to minimize heat transfer rates or to minimize operating cost, free convection is often preferred to forced convection.

There are, of course, many applications. Free convection strongly influences the operating temperatures of power generating and electronic devices. It plays a major role in a vast array of thermal manufacturing applications. Free convection is important in establishing temperature distributions within buildings and in determining heat losses or heat loads for heating, ventilating, and air conditioning systems. Free convection distributes the poisonous products of combustion during fires and is relevant to the environmental sciences, where it drives oceanic and atmospheric motions, as well as the related heat transfer and mass transfer processes.

In this chapter our objectives are to obtain an appreciation for the physical origins and nature of buoyancy-driven flows and to acquire tools for performing related heat transfer calculations.

9.1 Physical Considerations

In free convection fluid motion is due to buoyancy forces within the fluid, while in forced convection it is externally imposed. *Buoyancy is due to the combined presence of a fluid density gradient and a body force that is proportional to density.* In practice, the body force is usually *gravitational*, although it may be a centrifugal force in rotating fluid machinery or a Coriolis force in atmospheric and oceanic rotational motions. There are also several ways in which a mass density gradient may arise in a fluid, but for the most common situation it is due to the presence of a temperature gradient. We know that the density of gases and liquids depends on temperature, generally decreasing (due to fluid expansion) with increasing temperature ($\partial\rho/\partial T < 0$).

In this text we focus on free convection problems in which the density gradient is due to a temperature gradient and the body force is gravitational. However, the presence of a fluid density gradient in a gravitational field does not ensure the existence of free convection currents. Consider the conditions of Figure 9.1. A fluid is enclosed by two large, horizontal plates of different temperature ($T_1 \neq T_2$). In case *a* the temperature of the lower

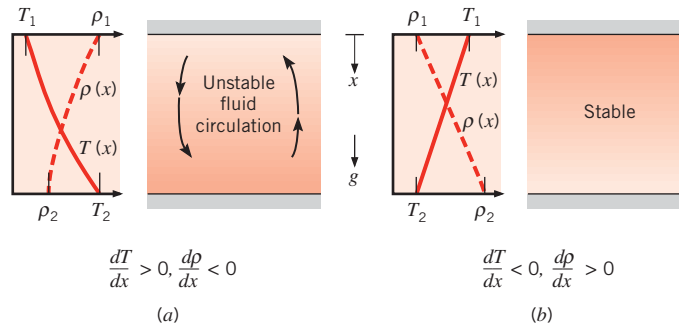


FIGURE 9.1 Conditions in a fluid between large horizontal plates at different temperatures: (a) Unstable temperature gradient. (b) Stable temperature gradient.

plate exceeds that of the upper plate, and the density decreases in the direction of the gravitational force. If the temperature difference exceeds a critical value, conditions are *unstable* and buoyancy forces are able to overcome the retarding influence of viscous forces. The gravitational force on the denser fluid in the upper layers exceeds that acting on the lighter fluid in the lower layers, and the designated circulation pattern will exist. The heavier fluid will descend, being warmed in the process, while the lighter fluid will rise, cooling as it moves. However, this condition does not characterize case *b*, for which $T_1 > T_2$ and the density no longer decreases in the direction of the gravitational force. Conditions are now *stable*, and there is no bulk fluid motion. In case *a* heat transfer occurs from the bottom to the top surface by free convection; for case *b* heat transfer (from top to bottom) occurs by conduction.

Free convection flows may be classified according to whether the flow is bounded by a surface. In the absence of an adjoining surface, *free boundary flows* may occur in the form of a *plume* or a *buoyant jet* (Figure 9.2). A plume is associated with fluid rising from a submerged hot object. Consider the hot wire of Figure 9.2a, which is immersed in an *extensive, quiescent*

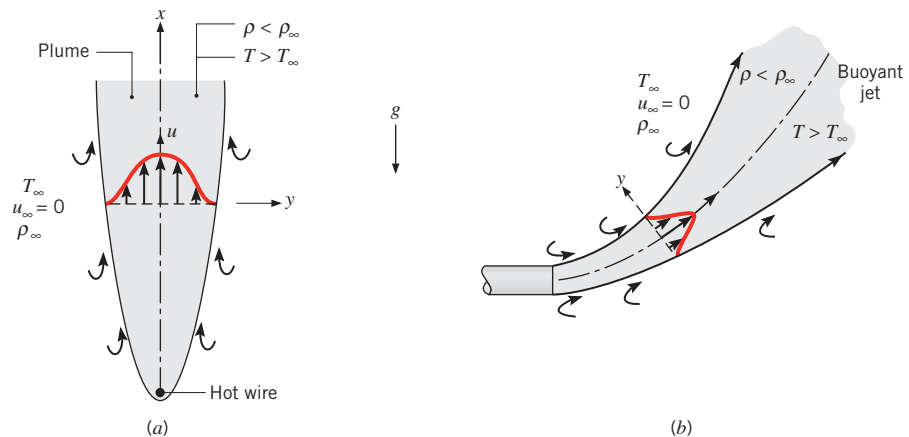


FIGURE 9.2 Buoyancy-driven free boundary layer flows in an extensive, quiescent medium. (a) Plume formation above a hot wire. (b) Buoyant jet associated with a hot discharge.

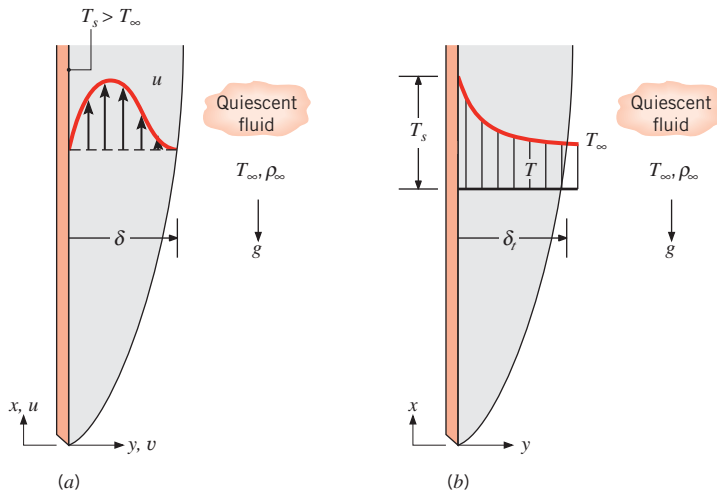


FIGURE 9.3 Boundary layer development on a hot vertical plate: (a) Velocity boundary layer. (b) Thermal boundary layer.

fluid.¹ Fluid that is heated by the wire rises due to buoyancy forces, entraining fluid from the quiescent region. Although the width of the plume increases with distance from the wire, the plume itself will eventually dissipate as a result of viscous effects and a reduction in the buoyancy force caused by cooling of the fluid in the plume. The distinction between a plume and a buoyant jet is generally made on the basis of the *initial* fluid velocity. This velocity is zero for the plume, but non-zero for the buoyant jet. Figure 9.2b shows a hot fluid being discharged as a horizontal jet into a quiescent medium of lower temperature. The vertical motion that the jet begins to assume is due to the buoyancy force. Such a condition occurs when warm water from the condenser of a central power station is discharged into a reservoir of cooler water. Free boundary flows are discussed in considerable detail by Jaluria [1] and Gebhart et al. [2].

In this text we focus on free convection flows bounded by a surface, and a classic example relates to boundary layer development on a hot vertical plate (Figure 9.3). The plate is immersed in an extensive, quiescent fluid, and with $T_s > T_\infty$ the fluid close to the plate is less dense than fluid that is further removed. Buoyancy forces therefore induce a free convection boundary layer in which the fluid that is heated by the plate rises vertically, entraining fluid from the quiescent region. The resulting velocity distribution is unlike that associated with forced convection boundary layers. In particular, the velocity is zero as $y \rightarrow \infty$, as well as at $y = 0$. A free convection boundary layer also develops if $T_s < T_\infty$. In this case, however, fluid motion is downward.

9.2 The Governing Equations for Laminar Boundary Layers

As for forced convection, the equations that describe momentum and energy transfer in free convection originate from the related conservation principles. Moreover, the specific processes are much like those that dominate in forced convection. Inertia and viscous forces remain important, as does energy transfer by advection and diffusion. The difference between the two flows is that, in free convection, a major role is played by buoyancy forces. Such forces, in fact, drive the flow.

¹An extensive medium is, in principle, an infinite medium. Since a quiescent fluid is one that is otherwise at rest, the velocity of fluid far from the hot wire is zero.

Consider a laminar boundary layer flow (Figure 9.3) that is driven by buoyancy forces. Assume steady, two-dimensional, constant property conditions in which the gravity force acts in the negative x -direction. Also, with one exception, assume the fluid to be incompressible. The exception involves accounting for the effect of variable density only in the buoyancy force, since it is this variation that induces fluid motion. Finally, assume that the boundary layer approximations of Section 6.4.1 are valid.

With the foregoing simplifications the x -momentum equation (Equation E.2) reduces to the boundary layer equation (Equation 6.28), except that the body force term X is retained. If the only contribution to this force is made by gravity, the body force per unit volume is $X = -\rho g$, where g is the local acceleration due to gravity. The appropriate form of the x -momentum equation is then

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{dp_\infty}{dx} - g + \nu \frac{\partial^2 u}{\partial y^2} \quad (9.1)$$

where dp_∞/dx is the free stream pressure gradient in the quiescent region *outside* the boundary layer. In this region, $u = 0$ and Equation 9.1 reduces to

$$\frac{dp_\infty}{dx} = -\rho_\infty g$$

Substituting this equation into 9.1, we obtain the following expression:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = g(\Delta\rho/\rho) + \nu \frac{\partial^2 u}{\partial y^2} \quad (9.2)$$

where $\Delta\rho = \rho_\infty - \rho$. This expression must apply at every point in the free convection boundary layer.

The first term on the right-hand side of Equation 9.2 is the buoyancy force per unit mass, and flow originates because the density ρ is a variable. If density variations are due only to temperature variations, the term may be related to a fluid property known as the *vol-umetric thermal expansion coefficient*

$$\beta = -\frac{1}{\rho} \left(\frac{\partial \rho}{\partial T} \right)_p \quad (9.3)$$

This *thermodynamic* property of the fluid provides a measure of the amount by which the density changes in response to a change in temperature at constant pressure. If it is expressed in the following approximate form,

$$\beta \approx -\frac{1}{\rho} \frac{\Delta\rho}{\Delta T} = -\frac{1}{\rho} \frac{\rho_\infty - \rho}{T_\infty - T}$$

it follows that

$$(\rho_\infty - \rho) \approx \rho\beta(T - T_\infty)$$

This simplification is known as the *Boussinesq approximation*, and substituting into Equation 9.2, the x -momentum equation becomes

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = g\beta(T - T_\infty) + \nu \frac{\partial^2 u}{\partial y^2} \quad (9.4)$$

where it is now apparent how the buoyancy force, which drives the flow, is related to the temperature difference.

Since buoyancy effects are confined to the momentum equation (9.4), the mass and energy conservation equations are unchanged from forced convection. Equations 6.27 and 6.29 may then be used to complete the problem formulation. The set of governing equations is then

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (9.5)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \quad (9.6)$$

and Equation 9.4. Note that viscous dissipation has been neglected in the energy equation, (9.6), an assumption that is certainly reasonable for the small velocities associated with free convection. In the mathematical sense the appearance of the buoyancy term in Equation 9.4 complicates matters. No longer may the hydrodynamic problem, given by Equations 9.4 and 9.5, be uncoupled from and solved to the exclusion of the thermal problem, given by Equation 9.6. The solution to the momentum equation depends on knowledge of T , and hence on the solution to the energy equation. Equations 9.4 through 9.6 are therefore strongly coupled and must be solved simultaneously.

As evident in Equation 9.4, the fluid's volumetric thermal expansion coefficient, β , plays a direct role in driving fluid motion in free convection. For an ideal gas, $\rho = p/RT$, and from Equation 9.3,

$$\beta = -\frac{1}{\rho} \left(\frac{\partial \rho}{\partial T} \right)_p = \frac{1}{\rho} \frac{p}{RT^2} = \frac{1}{T} \quad (9.7)$$

where T is the *absolute* temperature. For liquids and nonideal gases, β must be obtained from appropriate property tables (Appendix A).

9.3 Similarity Considerations

Let us now consider the dimensionless parameters that govern free convective flow and heat transfer for the vertical plate. As for forced convection (Chapter 6), the parameters may be obtained by nondimensionalizing the governing equations. Introducing

$$\begin{aligned} x^* &\equiv \frac{x}{L} & y^* &\equiv \frac{y}{L} \\ u^* &\equiv \frac{u}{u_0} & v^* &\equiv \frac{v}{u_0} & T^* &\equiv \frac{T - T_\infty}{T_s - T_\infty} \end{aligned}$$

where L is a characteristic length and u_0 is a reference velocity,² the x -momentum and energy equations (9.4 and 9.6) reduce to

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \frac{g\beta(T_s - T_\infty)L}{u_0^2} T^* + \frac{1}{Re_L} \frac{\partial^2 u^*}{\partial y^{*2}} \quad (9.8)$$

$$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{Re_L Pr} \frac{\partial^2 T^*}{\partial y^{*2}} \quad (9.9)$$

²Since free stream conditions are quiescent in free convection, there is no logical external reference velocity (V or u_∞), as in forced convection.

The dimensionless parameter in the first term on the right-hand side of Equation 9.8 is a direct consequence of the buoyancy force. The reference velocity u_0 can be specified to simplify the form of the equation. It is convenient to choose $u_0^2 = g\beta(T_s - T_\infty)L$, so that the term multiplying T^* becomes unity. Then, Re_L becomes $[g\beta(T_s - T_\infty)L^3/\nu^2]^{1/2}$. It is customary to define the *Grashof number* Gr_L as the square of this Reynolds number:

$$Gr_L \equiv \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} \quad (9.10)$$

As a result, Re_L in Equations 9.8 and 9.9 is replaced by $Gr_L^{1/2}$, and we see that the Grashof number (or more precisely, $Gr_L^{1/2}$) plays a role in free convection that is similar to the role played by the Reynolds number in forced convection. That is, while the Reynolds number is a measure of *inertial* forces relative to viscous forces, the Grashof number is a measure of *buoyancy* forces relative to viscous forces. We therefore expect heat transfer correlations for free convection to be of the form

$$\overline{Nu}_L = f(Gr_L, Pr) \quad (9.11)$$

With the exception of zero gravity conditions, inertial *and* buoyancy forces exist in every forced convection heat transfer scenario. Similarly, buoyancy *and* inertial forces will be present in the boundary layer of Figure 9.3 if the free stream velocity, u_∞ , is non-zero. In the case of Figure 9.3 with $u_\infty \neq 0$, it is convenient to choose u_∞ as the characteristic velocity, resulting in $u^*(y^* \rightarrow \infty) \rightarrow 1$. The T^* term in Equation 9.8 will then be multiplied by Gr_L/Re_L^2 , and the resulting Nusselt number expressions will be of the form

$$\overline{Nu}_L = f(Re_L, Gr_L, Pr) \quad (9.12)$$

In general, fluid motion may be driven primarily by inertial forces, primarily by buoyancy forces, or by a combination of inertial and buoyancy forces. The dominant effect is determined by the value of Gr_L/Re_L^2 , which is a measure of buoyancy forces relative to inertial forces. The various convection processes are summarized in Table 9.1. Note that, in Chapters 6 through 8, we implicitly assumed that the inequality $Gr_L/Re_L^2 \ll 1$ was satisfied. We will assume $Gr_L/Re_L^2 \gg 1$ in Sections 9.1 through 9.8. Discussion of combined (mixed) free and forced convection is provided in Section 9.9.

9.4 Laminar Free Convection on a Vertical Surface

Numerous solutions to the laminar free convection boundary layer equations have been obtained, and a special case that has received much attention involves free convection from

TABLE 9.1 Free, forced, and mixed convection processes, and the corresponding correlation forms

Process	Measure of buoyancy relative to inertial forces	Form of correlation	
Forced convection	$Gr_L/Re_L^2 \ll 1$	$\overline{Nu}_L = f(Re_L, Pr)$	(6.50)
Free convection	$Gr_L/Re_L^2 \gg 1$	$\overline{Nu}_L = f(Gr_L, Pr)$	(9.11)
Mixed convection	$Gr_L/Re_L^2 \approx 1$	$\overline{Nu}_L = f(Re_L, Gr_L, Pr)$	(9.12)

an isothermal vertical surface in an extensive quiescent medium (Figure 9.3). For this geometry Equations 9.4 through 9.6 must be solved subject to boundary conditions of the form³

$$\begin{aligned} y = 0: & \quad u = v = 0 & \quad T = T_s \\ y \rightarrow \infty: & \quad u \rightarrow 0 & \quad T \rightarrow T_\infty \end{aligned}$$

A similarity solution to the foregoing problem has been obtained by Ostrach [3]. The solution involves transforming variables by introducing a similarity variable of the form

$$\eta \equiv \frac{y}{x} \left(\frac{Gr_x}{4} \right)^{1/4} \quad (9.13)$$

and representing the velocity components in terms of a stream function defined as

$$\psi(x, y) \equiv f(\eta) \left[4\nu \left(\frac{Gr_x}{4} \right)^{1/4} \right] \quad (9.14)$$

With the foregoing definition of the stream function, the x -velocity component may be expressed as

$$\begin{aligned} u &= \frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial \eta} \frac{\partial \eta}{\partial y} = 4\nu \left(\frac{Gr_x}{4} \right)^{1/4} f'(\eta) \frac{1}{x} \left(\frac{Gr_x}{4} \right)^{1/4} \\ &= \frac{2\nu}{x} Gr_x^{1/2} f'(\eta) \end{aligned} \quad (9.15)$$

where primed quantities indicate differentiation with respect to η . Hence $f'(\eta) \equiv df/d\eta$. Evaluating the y -velocity component $v = -\partial\psi/\partial x$ in a similar fashion and introducing the dimensionless temperature

$$T^* \equiv \frac{T - T_\infty}{T_s - T_\infty} \quad (9.16)$$

the three original partial differential equations (9.4 through 9.6) may then be reduced to two ordinary differential equations of the form

$$f''' + 3ff'' - 2(f')^2 + T^* = 0 \quad (9.17)$$

$$T^{*''} + 3PrfT^{*'} = 0 \quad (9.18)$$

where f and T^* are functions only of η and the double and triple primes, respectively, refer to second and third derivatives with respect to η . Note that f is the key dependent variable for the velocity boundary layer and that the continuity equation (9.5) is automatically satisfied through introduction of the stream function.

The transformed boundary conditions required to solve the momentum and energy equations (9.17 and 9.18) are of the form

$$\begin{aligned} \eta = 0: & \quad f = f' = 0 & \quad T^* = 1 \\ \eta \rightarrow \infty: & \quad f' \rightarrow 0 & \quad T^* \rightarrow 0 \end{aligned}$$

³The boundary layer approximations are assumed in using Equations 9.4 through 9.6. However, the approximations are only valid for $(Gr_x Pr) \gtrsim 10^4$. Below this value (close to the leading edge), the boundary layer thickness is too large relative to the characteristic length x to ensure the validity of the approximations.

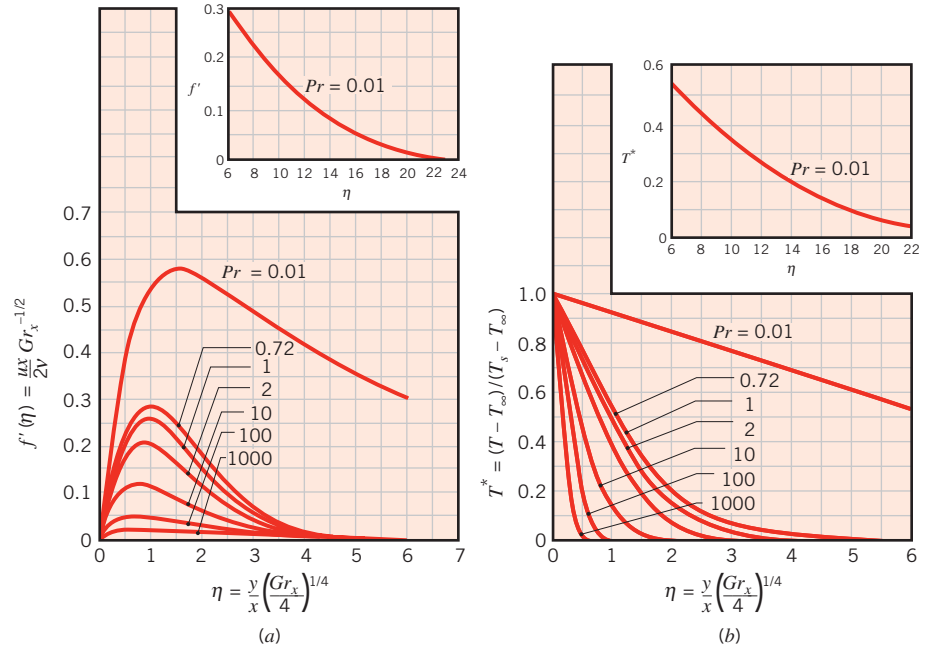


FIGURE 9.4 Laminar, free convection boundary layer conditions on an isothermal, vertical surface. (a) Velocity profiles. (b) Temperature profiles. Adapted from Ostrach, S., NACA Report 1111, 1953.

A numerical solution has been obtained by Ostrach [3], and selected results are shown in Figure 9.4. Values of the x -velocity component u and the temperature T at any value of x and y may be obtained from Figure 9.4a and Figure 9.4b, respectively.

Figure 9.4b may also be used to infer the appropriate form of the heat transfer correlation. Using Newton's law of cooling for the local convection coefficient h , the local Nusselt number may be expressed as

$$Nu_x = \frac{hx}{k} = \frac{[q_s''/(T_s - T_\infty)]x}{k}$$

Using Fourier's law to obtain q_s'' and expressing the surface temperature gradient in terms of η , Equation 9.13, and T^* , Equation 9.16, it follows that

$$q_s'' = -k \frac{\partial T}{\partial y} \Big|_{y=0} = -\frac{k}{x} (T_s - T_\infty) \left(\frac{Gr_x}{4} \right)^{1/4} \frac{dT^*}{d\eta} \Big|_{\eta=0}$$

Hence

$$Nu_x = \frac{hx}{k} = - \left(\frac{Gr_x}{4} \right)^{1/4} \frac{dT^*}{d\eta} \Big|_{\eta=0} = \left(\frac{Gr_x}{4} \right)^{1/4} g(Pr) \quad (9.19)$$

which acknowledges that the dimensionless temperature gradient at the surface is a function of the Prandtl number $g(Pr)$. This dependence is evident from Figure 9.4b and has

been determined numerically for selected values of Pr [3]. The results have been correlated to within 0.5% by an interpolation formula of the form [4]

$$g(Pr) = \frac{0.75 Pr^{1/2}}{(0.609 + 1.221 Pr^{1/2} + 1.238 Pr)^{1/4}} \quad (9.20)$$

which applies for $0 \leq Pr \leq \infty$.

Using Equation 9.19 for the local convection coefficient and substituting for the local Grashof number,

$$Gr_x = \frac{g\beta(T_s - T_\infty)x^3}{\nu^2}$$

the average convection coefficient for a surface of length L is then

$$\bar{h} = \frac{1}{L} \int_0^L h dx = \frac{k}{L} \left[\frac{g\beta(T_s - T_\infty)}{4\nu^2} \right]^{1/4} g(Pr) \int_0^L \frac{dx}{x^{1/4}}$$

Integrating, it follows that

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = \frac{4}{3} \left(\frac{Gr_L}{4} \right)^{1/4} g(Pr) \quad (9.21)$$

or substituting from Equation 9.19, with $x = L$,

$$\overline{Nu}_L = \frac{4}{3} Nu_L \quad (9.22)$$

The foregoing results apply irrespective of whether $T_s > T_\infty$ or $T_s < T_\infty$. If $T_s < T_\infty$, conditions are inverted from those of Figure 9.3. The leading edge is at the top of the plate, and positive x is defined in the direction of the gravity force.

9.5 The Effects of Turbulence

It is important to note that free convection boundary layers are not restricted to laminar flow. As with forced convection, *hydrodynamic instabilities* may arise. That is, disturbances in the flow may be amplified, leading to transition from laminar to turbulent flow. This process is shown schematically in Figure 9.5 for a hot vertical plate.

Transition in a free convection boundary layer depends on the relative magnitude of the buoyancy and viscous forces in the fluid. It is customary to correlate its occurrence in

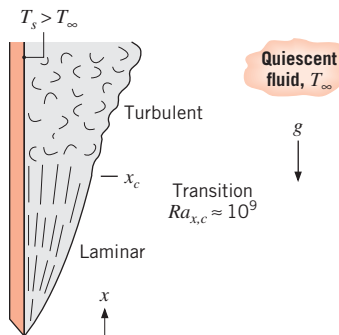


FIGURE 9.5 Free convection boundary layer transition on a vertical plate.

terms of the *Rayleigh number*, which is simply the product of the Grashof and Prandtl numbers. For vertical plates the critical Rayleigh number is

$$Ra_{x,c} = Gr_{x,c} Pr = \frac{g\beta(T_s - T_\infty)x^3}{\nu\alpha} \approx 10^9 \quad (9.23)$$

An extensive discussion of stability and transition effects is provided by Gebhart et al. [2].

As in forced convection, transition to turbulence has a strong effect on heat transfer. Hence the results of the foregoing section apply only if $Ra_L \lesssim 10^9$. To obtain appropriate correlations for turbulent flow, emphasis is placed on experimental results.

EXAMPLE 9.1

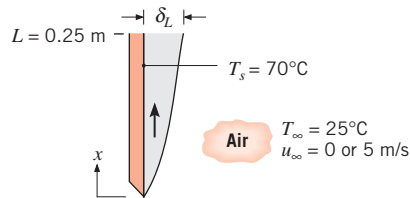
Consider a 0.25-m-long vertical plate that is at 70°C. The plate is suspended in quiescent air that is at 25°C. Estimate the velocity boundary layer thickness and maximum upward velocity at the trailing edge of the plate. How does the boundary layer thickness compare with the thickness that would exist if the air were flowing over the plate at a free stream velocity of 5 m/s?

SOLUTION

Known: Vertical plate is in quiescent air at a lower temperature.

Find: Velocity boundary layer thickness and maximum upward velocity at trailing edge. Compare boundary layer thickness with value corresponding to an air speed of 5 m/s.

Schematic:



Assumptions:

1. Ideal gas.
2. Constant properties.

Properties: Table A.4, air ($T_f = 320.5 \text{ K}$): $\nu = 17.95 \times 10^{-6} \text{ m}^2/\text{s}$, $Pr = 0.7$, $\beta = T_f^{-1} = 3.12 \times 10^{-3} \text{ K}^{-1}$.

Analysis: Equation 9.10 gives

$$\begin{aligned} Gr_L &= \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} \\ &= \frac{9.8 \text{ m/s}^2 \times (3.12 \times 10^{-3} \text{ K}^{-1})(70 - 25)^\circ\text{C}(0.25 \text{ m})^3}{(17.95 \times 10^{-6} \text{ m}^2/\text{s})^2} = 6.69 \times 10^7 \end{aligned}$$

Hence $Ra_L = Gr_L Pr = 4.68 \times 10^7$ and, from Equation 9.23, the free convection boundary layer is laminar. The analysis of Section 9.4 is therefore applicable. From the results of Figure 9.4a, it follows that, for $Pr = 0.7$, $\eta \approx 6.0$ at the edge of the boundary layer, that is, at $y \approx \delta$. Hence

$$\delta_L \approx \frac{6L}{(Gr_L/4)^{1/4}} = \frac{6(0.25 \text{ m})}{(1.67 \times 10^7)^{1/4}} = 0.024 \text{ m} \quad \triangleleft$$

From Figure 9.4a, it can be seen that the maximum velocity corresponds to $f'(\eta) \approx 0.28$ and the velocity is

$$u = \frac{2\nu f'(\eta) Gr_L^{1/2}}{L} \approx \frac{2 \times 17.95 \times 10^{-6} \text{ m}^2/\text{s} \times 0.28 \times (6.69 \times 10^7)^{1/2}}{0.25 \text{ m}} = 0.33 \text{ m/s} \quad \triangleleft$$

For airflow at $u_\infty = 5 \text{ m/s}$

$$Re_L = \frac{u_\infty L}{\nu} = \frac{(5 \text{ m/s}) \times 0.25 \text{ m}}{17.95 \times 10^{-6} \text{ m}^2/\text{s}} = 6.97 \times 10^4$$

and $Gr_L/Re_L^2 = 6.69 \times 10^7 / (6.97 \times 10^4)^2 = 0.014 \ll 1$ so an analysis using the results of Chapter 7 is applicable. Since $Re_L < 5 \times 10^5$, the boundary layer is laminar, and from Equation 7.19,

$$\delta_L \approx \frac{5L}{Re_L^{1/2}} = \frac{5(0.25 \text{ m})}{(6.97 \times 10^4)^{1/2}} = 0.0047 \text{ m} \quad \triangleleft$$

Comments: Free convection boundary layers typically have smaller velocities than in forced convection, which leads to thicker boundary layers. In turn, free convection boundary layers typically pose a larger resistance to heat transfer than forced convection boundary layers.

9.6 Empirical Correlations: External Free Convection Flows

In the preceding sections, we considered free convection associated with laminar boundary layer development adjacent to a hot vertical plate and transition of the laminar flow to a turbulent state. In doing so, we introduced two dimensionless parameters, the Grashof number Gr and the Rayleigh number Ra , which also appear in empirical correlations for free convection involving both laminar and turbulent flow conditions and in geometries other than a flat plate.

In this section we summarize empirical correlations that have been developed for common *immersed* (external flow) geometries. The correlations are suitable for many engineering calculations and are often of the form

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = C Ra_L^n \quad (9.24)$$

where the Rayleigh number,

$$Ra_L = Gr_L Pr = \frac{g\beta(T_s - T_\infty)L^3}{\nu\alpha} \quad (9.25)$$

is based on the characteristic length L of the geometry. Typically, $n = \frac{1}{4}$ and $\frac{1}{3}$ for laminar and turbulent flows, respectively. For turbulent flow it then follows that $\bar{h}_L$ is independent of L . Note that all properties are evaluated at the film temperature, $T_f \equiv (T_s + T_\infty)/2$.

9.6.1 The Vertical Plate

Expressions of the form given by Equation 9.24 have been developed for the vertical plate [5–7]. For laminar flow ($10^4 \lesssim Ra_L \lesssim 10^9$), $C = 0.59$ and $n = 1/4$, and for combined laminar and turbulent flow ($10^9 \lesssim Ra_L \lesssim 10^{13}$), $C = 0.10$ and $n = 1/3$. A correlation that may be applied over the *entire* range of Ra_L has been recommended by Churchill and Chu [8] and is of the form

$$\overline{Nu}_L = \left\{ 0.825 + \frac{0.387 Ra_L^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}} \right\}^2 \quad (9.26)$$

where L is the plate length. Although Equation 9.26 is suitable for most engineering calculations, slightly better accuracy may be obtained for laminar flow by using [8]

$$\overline{Nu}_L = 0.68 + \frac{0.670 Ra_L^{1/4}}{[1 + (0.492/Pr)^{9/16}]^{4/9}} \quad Ra_L \lesssim 10^9 \quad (9.27)$$

When the Rayleigh number is moderately large, the second term on the right-hand side of Equations 9.26 and 9.27 dominates, and the correlations are the same form as Equation 9.24, except that the constant, C , is replaced by a function of Pr . Equation 9.27 is then in excellent quantitative agreement with the analytical solution given by Equations 9.21 and 9.20. In contrast, when the Rayleigh number is small, the first term on the right-hand side of Equations 9.26 and 9.27 dominates, and the equations yield the same behavior since $0.825^2 \approx 0.68$. The presence of leading constants in Equations 9.26 and 9.27 accounts for the fact that, for small Rayleigh number, the boundary layer assumptions become invalid and conduction parallel to the plate is important.

It is important to recognize that the foregoing results have been obtained for an isothermal plate (constant T_s). If the surface condition is, instead, one of uniform heat flux (constant q_s''), the temperature difference $(T_s - T_\infty)$ will vary with x , increasing from the leading edge. An approximate procedure for determining this variation may be based on results [8, 9] showing that $\overline{Nu}_L$ correlations obtained for the isothermal plate may still be used to an excellent approximation, if $\overline{Nu}_L$ and Ra_L are defined in terms of the temperature difference at the midpoint of the plate, $\Delta T_{L/2} = T_s(L/2) - T_\infty$. Hence, with $\bar{h} \equiv q_s''/\Delta T_{L/2}$, a correlation such as Equation 9.27 could be used to determine $\Delta T_{L/2}$ (for example, using a trial-and-error technique), and hence the midpoint surface temperature $T_s(L/2)$. If it is assumed that $Nu_x \propto Ra_x^{1/4}$ over the entire plate, it follows that

$$\frac{q_s''x}{k\Delta T} \propto \Delta T^{1/4} x^{3/4}$$

or

$$\Delta T \propto x^{1/5}$$

Hence the temperature difference at any x is

$$\Delta T_x \approx \frac{x^{1/5}}{(L/2)^{1/5}} \Delta T_{L/2} = 1.15 \left(\frac{x}{L} \right)^{1/5} \Delta T_{L/2} \quad (9.28)$$

A more detailed discussion of constant heat flux results is provided by Churchill [10].

The foregoing results may also be applied to *vertical* cylinders of height L , if the boundary layer thickness δ is much less than the cylinder diameter D . This condition is known to be satisfied [11] when

$$\frac{D}{L} \gtrsim \frac{35}{Gr_L^{1/4}}$$

Cebeci [12] and Minkowycz and Sparrow [13] present results for slender, vertical cylinders not meeting this condition, where transverse curvature influences boundary layer development and enhances the rate of heat transfer.

EXAMPLE 9.2

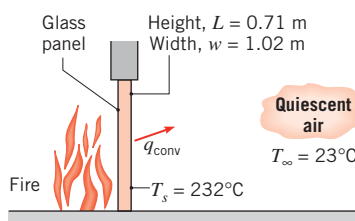
A glass-door firescreen, used to reduce exfiltration of room air through a chimney, has a height of 0.71 m and a width of 1.02 m and reaches a temperature of 232°C. If the room temperature is 23°C, estimate the convection heat rate from the fireplace to the room.

SOLUTION

Known: Glass screen situated in fireplace opening.

Find: Rate of heat transfer by convection between screen and room air.

Schematic:



Assumptions:

1. Screen is at a uniform temperature T_s .
2. Room air is quiescent.
3. Ideal gas.
4. Constant properties.

Properties: Table A.4, air ($T_f = 400$ K): $k = 33.8 \times 10^{-3}$ W/m · K, $\nu = 26.4 \times 10^{-6}$ m²/s, $\alpha = 38.3 \times 10^{-6}$ m²/s, $Pr = 0.690$, $\beta = (1/T_f) = 0.0025$ K⁻¹.

Analysis: The rate of heat transfer by free convection from the panel to the room is given by Newton's law of cooling

$$q = \bar{h}A_s(T_s - T_\infty)$$

where $\bar{h}$ may be obtained from knowledge of the Rayleigh number. Using Equation 9.25,

$$\begin{aligned} Ra_L &= \frac{g\beta(T_s - T_\infty)L^3}{\alpha\nu} \\ &= \frac{9.8 \text{ m/s}^2 \times 0.0025 \text{ K}^{-1} \times (232 - 23)^\circ\text{C} \times (0.71 \text{ m})^3}{38.3 \times 10^{-6} \text{ m}^2/\text{s} \times 26.4 \times 10^{-6} \text{ m}^2/\text{s}} = 1.813 \times 10^9 \end{aligned}$$

and from Equation 9.23 it follows that transition to turbulence occurs on the panel. The appropriate correlation is then given by Equation 9.26

$$\begin{aligned} \overline{Nu}_L &= \left\{ 0.825 + \frac{0.387 Ra_L^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}} \right\}^2 \\ \overline{Nu}_L &= \left\{ 0.825 + \frac{0.387(1.813 \times 10^9)^{1/6}}{[1 + (0.492/0.690)^{9/16}]^{8/27}} \right\}^2 = 147 \end{aligned}$$

Hence

$$\bar{h} = \frac{\overline{Nu}_L \cdot k}{L} = \frac{147 \times 33.8 \times 10^{-3} \text{ W/m} \cdot \text{K}}{0.71 \text{ m}} = 7.0 \text{ W/m}^2 \cdot \text{K}$$

and

$$q = 7.0 \text{ W/m}^2 \cdot \text{K}(1.02 \times 0.71) \text{ m}^2(232 - 23)^\circ\text{C} = 1060 \text{ W}$$

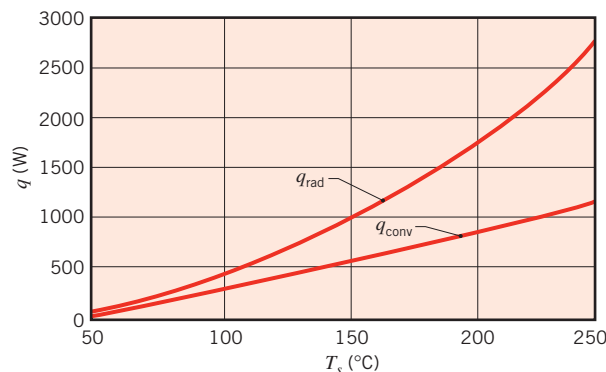
Comments:

1. Radiation heat transfer effects are often significant relative to free convection. Using Equation 1.7 and assuming $\varepsilon = 1.0$ for the glass surface and $T_{\text{sur}} = 23^\circ\text{C}$, the net rate of radiation heat transfer between the glass and the surroundings is

$$\begin{aligned} q_{\text{rad}} &= \varepsilon A_s \sigma (T_s^4 - T_{\text{sur}}^4) \\ q_{\text{rad}} &= 1(1.02 \times 0.71) \text{ m}^2 \times 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 (505^4 - 296^4) \text{ K}^4 \\ q_{\text{rad}} &= 2355 \text{ W} \end{aligned}$$

Hence in this case radiation heat transfer exceeds free convection heat transfer by more than a factor of 2.

2. The effects of radiation and free convection on heat transfer from the glass depend strongly on its temperature. With $q \propto T_s^4$ for radiation and $q \propto T_s^n$ for free convection, where $1.25 < n < 1.33$, we expect the relative influence of radiation to increase with increasing temperature. This behavior is revealed by computing and plotting the heat rates as a function of temperature for $50 \leq T_s \leq 250^\circ\text{C}$.



For each value of T_s used to generate the foregoing free convection results, air properties were determined at the corresponding value of T_f .

9.6.2 Inclined and Horizontal Plates

For a vertical plate that is hot (or cold) relative to an ambient fluid, the plate is aligned with the gravitational vector, and the buoyancy force acts exclusively to induce fluid motion in the upward (or downward) direction. However, if the plate is inclined with respect to gravity, the buoyancy force has a component normal, as well as parallel, to the plate surface. With a reduction in the buoyancy force parallel to the surface, there is a reduction in fluid velocities along the plate, and one might expect there to be an attendant reduction in convection heat transfer. Whether, in fact, there is such a reduction depends on whether one is interested in heat transfer from the top or bottom surface of the plate.

As shown in Figure 9.6a, if the plate is cold, the y-component of the buoyancy force, which is normal to the plate, acts to maintain the descending boundary layer flow in contact with the top surface of the plate. Since the x-component of the gravitational acceleration is reduced to $g \cos \theta$, fluid velocities along the plate are reduced and there is an attendant reduction in convection heat transfer to the top surface. However, at the bottom surface, the y-component of the buoyancy force acts to move fluid from the surface, and boundary layer development is interrupted by the discharge of parcels of cool fluid from the surface (Figure 9.6a). The resulting flow is three-dimensional, and, as shown by the spanwise (z -direction) variations of Figure 9.6b, the cool fluid discharged from the bottom surface is continuously replaced by the warmer ambient fluid. The displacement of cool boundary layer fluid by the warmer ambient and the attendant reduction in the thermal boundary layer thickness act to increase convection heat transfer to the bottom surface. In fact, heat transfer enhancement due to the three-dimensional flow typically exceeds the reduction associated with the reduced x-component of g , and the combined effect is to increase heat transfer to the bottom surface. Similar trends characterize a hot plate (Figure 9.6c,d), and the three-dimensional flow is now associated with the upper surface, from which parcels of warm fluid are discharged. Such flows have been observed by several investigators [14–16].

In an early study of heat transfer from inclined plates, Rich [17] suggested that convection coefficients could be determined from vertical plate correlations, if g is replaced by $g \cos \theta$ in computing the plate Rayleigh number. Since then, however, it has been determined that this approach is only satisfactory for the top and bottom surfaces of cold and hot plates, respectively. It is not appropriate for the top and bottom surfaces of hot and cold plates,

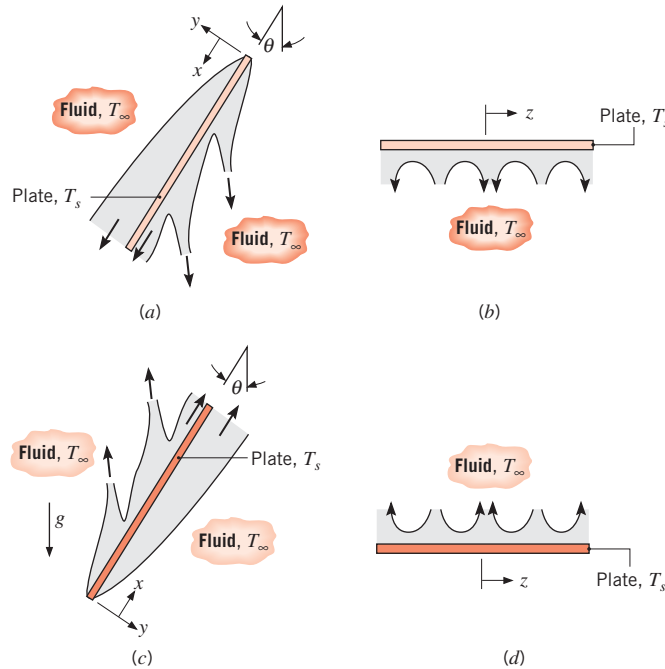


FIGURE 9.6 Buoyancy-driven flows on an inclined plate: (a) Side view of flows at top and bottom surfaces of a cold plate ($T_s < T_\infty$). (b) End view of flow at bottom surface of cold plate. (c) Side view of flows at top and bottom surfaces of a hot plate ($T_s > T_\infty$). (d) End view of flow at top surface of hot plate.

respectively, where the three-dimensionality of the flow has limited the ability to develop generalized correlations. At the top and bottom surfaces of cold and hot inclined plates, respectively, it is therefore recommended that, for $0 \leq \theta \lesssim 60^\circ$, g be replaced by $g \cos \theta$ and that Equation 9.26 or 9.27 be used to compute the average Nusselt number. For the opposite surfaces, no recommendations are made, and the literature should be consulted [14–16].

If the plate is horizontal, the buoyancy force is exclusively normal to the surface. As for the inclined plate, flow patterns and heat transfer depend strongly on whether the surface is cold or hot and on whether it is facing upward or downward. For a cold surface facing upward (Figure 9.7a) and a hot surface facing downward (Figure 9.7d), the tendency of the fluid to descend and ascend, respectively, is impeded by the plate. The flow must move horizontally before it can descend or ascend from the edges of the plate, and convection heat transfer is somewhat ineffective. In contrast, for a cold surface facing downward (Figure 9.7b) and a hot surface facing upward (Figure 9.7c), flow is driven by descending and ascending parcels of fluid, respectively. Conservation of mass dictates that cold (warm) fluid descending (ascending) from a surface be replaced by ascending (descending) warmer (cooler) fluid from the ambient, and heat transfer is much more effective.

For horizontal plates of various shapes (for example, squares, rectangles, or circles), there is a need to define the characteristic length for use in the Nusselt and Rayleigh numbers. Experiments have shown [18, 19] that a single set of correlations can be used for a variety of different plate shapes when the characteristic length is defined as

$$L \equiv \frac{A_s}{P} \quad (9.29)$$

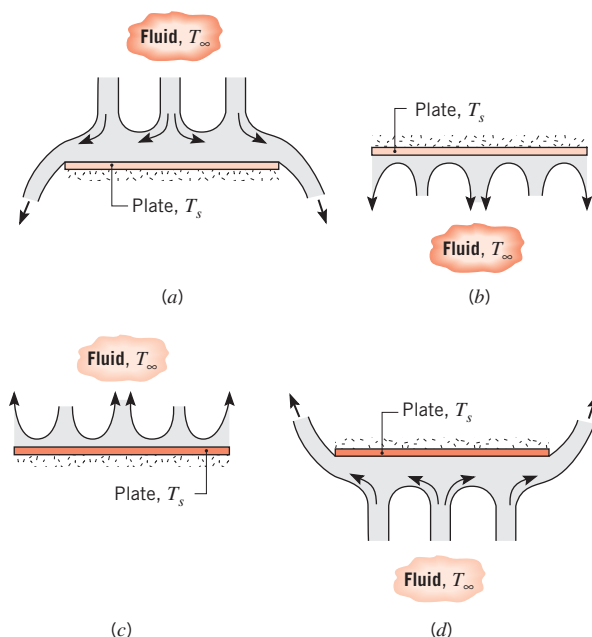


FIGURE 9.7 Buoyancy-driven flows on horizontal cold ($T_s < T_\infty$) and hot ($T_s > T_\infty$) plates: (a) Top surface of cold plate. (b) Bottom surface of cold plate. (c) Top surface of hot plate. (d) Bottom surface of hot plate.

where A_s and P are the plate surface area (one side) and perimeter, respectively. Using this characteristic length, the recommended correlations for the average Nusselt number are

Upper Surface of Hot Plate or Lower Surface of Cold Plate [19]:

$$\overline{Nu}_L = 0.54 Ra_L^{1/4} \quad (10^4 \lesssim Ra_L \lesssim 10^7, Pr \gtrsim 0.7) \quad (9.30)$$

$$\overline{Nu}_L = 0.15 Ra_L^{1/3} \quad (10^7 \lesssim Ra_L \lesssim 10^{11}, \text{all } Pr) \quad (9.31)$$

Lower Surface of Hot Plate or Upper Surface of Cold Plate [20]:

$$\overline{Nu}_L = 0.52 Ra_L^{1/5} \quad (10^4 \lesssim Ra_L \lesssim 10^9, Pr \gtrsim 0.7) \quad (9.32)$$

Additional correlations can be found in [21].

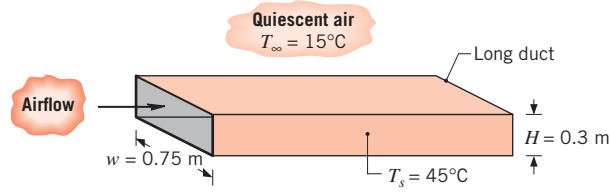
EXAMPLE 9.3

Airflow through a long rectangular heating duct that is 0.75 m wide and 0.3 m high maintains the outer duct surface at 45°C. If the duct is uninsulated and exposed to air at 15°C in the crawlspace beneath a home, what is the rate of heat loss from the duct per meter of length?

SOLUTION

Known: Surface temperature of a long rectangular duct.

Find: Rate of heat loss from duct per meter of length.

Schematic:**Assumptions:**

1. Ambient air is quiescent.
2. Surface radiation effects are negligible.
3. Ideal gas.
4. Constant properties.

Properties: Table A.4, air ($T_f = 303$ K): $\nu = 16.2 \times 10^{-6}$ m²/s, $\alpha = 22.9 \times 10^{-6}$ m²/s, $k = 0.0265$ W/m · K, $\beta = 0.0033$ K⁻¹, $Pr = 0.71$.

Analysis: Surface heat loss is by free convection from the vertical sides and the horizontal top and bottom. From Equation 9.25

$$Ra_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu\alpha} = \frac{(9.8 \text{ m/s}^2)(0.0033 \text{ K}^{-1})(30 \text{ K}) L^3 (\text{m}^3)}{(16.2 \times 10^{-6} \text{ m}^2/\text{s})(22.9 \times 10^{-6} \text{ m}^2/\text{s})}$$

$$Ra_L = 2.62 \times 10^9 L^3$$

For the two sides, $L = H = 0.3$ m. Hence $Ra_L = 7.07 \times 10^7$. The free convection boundary layer is therefore laminar, and from Equation 9.27

$$\overline{Nu}_L = 0.68 + \frac{0.670 Ra_L^{1/4}}{[1 + (0.492/Pr)^{9/16}]^{4/9}}$$

The convection coefficient associated with the sides is then

$$\bar{h}_s = \frac{k}{H} \overline{Nu}_L$$

$$\bar{h}_s = \frac{0.0265 \text{ W/m} \cdot \text{K}}{0.3 \text{ m}} \left\{ 0.68 + \frac{0.670(7.07 \times 10^7)^{1/4}}{[1 + (0.492/0.71)^{9/16}]^{4/9}} \right\} = 4.23 \text{ W/m}^2 \cdot \text{K}$$

For the top and bottom, $L = (A_s/P) \approx (w/2) = 0.375$ m. Hence $Ra_L = 1.38 \times 10^8$, and from Equations 9.31 and 9.32, respectively,

$$\bar{h}_t = [k/(w/2)] \times 0.15 Ra_L^{1/3} = \frac{0.0265 \text{ W/m} \cdot \text{K}}{0.375 \text{ m}} \times 0.15(1.38 \times 10^8)^{1/3}$$

$$= 5.47 \text{ W/m}^2 \cdot \text{K}$$

$$\bar{h}_b = [k/(w/2)] \times 0.52 Ra_L^{1/5} = \frac{0.0265 \text{ W/m} \cdot \text{K}}{0.375 \text{ m}} \times 0.52(1.38 \times 10^8)^{1/5}$$

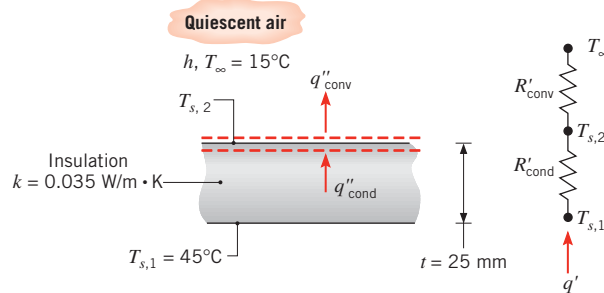
$$= 1.56 \text{ W/m}^2 \cdot \text{K}$$

The rate of heat loss per unit length of duct is then

$$\begin{aligned} q' &= 2q'_s + q'_t + q'_b = (2\bar{h}_s \cdot H + \bar{h}_t \cdot w + \bar{h}_b \cdot w)(T_s - T_\infty) \\ q' &= (2 \times 4.23 \times 0.3 + 5.47 \times 0.75 + 1.56 \times 0.75)(45 - 15) \text{ W/m} \\ q' &= 234 \text{ W/m} \end{aligned}$$

Comments:

1. The heat loss may be reduced by insulating the duct. We consider this option for a 25-mm-thick layer of blanket insulation ($k = 0.035 \text{ W/m} \cdot \text{K}$) that is wrapped around the duct.



The rate of heat loss at each surface may be expressed as

$$q' = \frac{T_{s,1} - T_\infty}{R'_{\text{cond}} + R'_{\text{conv}}}$$

where R'_{conv} is associated with free convection from the outer surface and hence depends on the unknown temperature $T_{s,2}$. This temperature may be determined by applying an energy balance to the outer surface, from which it follows that

$$q''_{\text{cond}} = q''_{\text{conv}}$$

or

$$\frac{(T_{s,1} - T_{s,2})}{(t/k)} = \frac{(T_{s,2} - T_\infty)}{(1/\bar{h})}$$

Since different convection coefficients are associated with the sides, top, and bottom ($\bar{h}_s$, $\bar{h}_t$, and $\bar{h}_b$), a separate solution to this equation must be obtained for each of the three surfaces. The solutions are iterative, since the properties of air and the convection coefficients depend on T_s . Performing the calculations, we obtain

Sides	$T_{s,2} = 24^\circ\text{C}$,	$\bar{h}_s = 3.18 \text{ W/m}^2 \cdot \text{K}$
Top	$T_{s,2} = 23^\circ\text{C}$,	$\bar{h}_t = 3.66 \text{ W/m}^2 \cdot \text{K}$
Bottom	$T_{s,2} = 30^\circ\text{C}$,	$\bar{h}_b = 1.36 \text{ W/m}^2 \cdot \text{K}$

Neglecting heat loss through the corners of the insulation, the total heat rate per unit length of duct is then

$$\begin{aligned} q' &= 2q'_s + q'_t + q'_b \\ q' &= \frac{2H(T_{s,1} - T_\infty)}{(t/k) + (1/\bar{h}_s)} + \frac{w(T_{s,1} - T_\infty)}{(t/k) + (1/\bar{h}_t)} + \frac{w(T_{s,1} - T_\infty)}{(t/k) + (1/\bar{h}_b)} \end{aligned}$$

which yields

$$q' = (17.5 + 22.8 + 15.5) \text{ W/m} = 55.8 \text{ W/m}$$

The insulation therefore provides a 76% reduction in heat loss to the ambient air by natural convection.

2. Although they have been neglected, radiation losses may still be significant. From Equation 1.7 with ε assumed to be unity and $T_{\text{sur}} = 288 \text{ K}$, $q'_{\text{rad}} = 398 \text{ W/m}$ for the uninsulated duct. Inclusion of radiation effects in the energy balance for the insulated duct would reduce the outer surface temperatures, thereby reducing the convection heat rates. With radiation, however, the total heat rate ($q'_{\text{conv}} + q'_{\text{rad}}$) would increase.

9.6.3 The Long Horizontal Cylinder

This important geometry has been studied extensively, and many existing correlations have been reviewed by Morgan [22]. For an isothermal cylinder, Morgan suggests an expression of the form

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = C Ra_D^n \quad (9.33)$$

where C and n are given in Table 9.2 and Ra_D and $\overline{Nu}_D$ are based on the cylinder diameter. In contrast, Churchill and Chu [23] have recommended a single correlation for a wide Rayleigh number range:

$$\overline{Nu}_D = \left\{ 0.60 + \frac{0.387 Ra_D^{1/6}}{[1 + (0.559/Pr)^{9/16}]^{8/27}} \right\}^2 \quad Ra_D \lesssim 10^{12} \quad (9.34)$$

The foregoing correlations provide the average Nusselt number over the entire circumference of an isothermal cylinder. As shown in Figure 9.8 for a hot cylinder, local Nusselt numbers are influenced by boundary layer development, which begins at $\theta = 0$ and concludes at $\theta < \pi$ with formation of a plume ascending from the cylinder. If the flow remains laminar over the entire surface, the distribution of the local Nusselt number with θ is characterized by a maximum at $\theta = 0$ and a monotonic decay with increasing θ . This decay would be disrupted at Rayleigh numbers sufficiently large ($Ra_D \gtrsim 10^9$) to permit transition to turbulence within the boundary layer. If the cylinder is cold relative to the ambient fluid, boundary layer development begins at $\theta = \pi$, the local Nusselt number is a maximum at this location, and the plume descends from the cylinder.

TABLE 9.2 Constants of Equation 9.33 for free convection on a horizontal circular cylinder [22]

Ra_D	C	n
10^{-10} – 10^{-2}	0.675	0.058
10^{-2} – 10^2	1.02	0.148
10^2 – 10^4	0.850	0.188
10^4 – 10^7	0.480	0.250
10^7 – 10^{12}	0.125	0.333

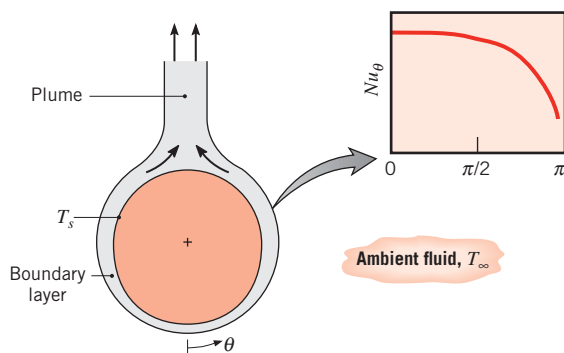


FIGURE 9.8 Boundary layer development and Nusselt number distribution on a hot horizontal cylinder.

EXAMPLE 9.4

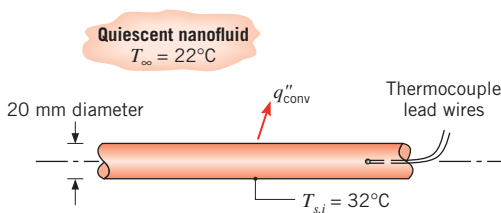
The fluid of Example 2.2 is characterized by a thermal conductivity, density, specific heat, and dynamic viscosity of $0.705 \text{ W/m} \cdot \text{K}$, 1146 kg/m^3 , $3587 \text{ J/kg} \cdot \text{K}$, and $962 \times 10^{-6} \text{ N} \cdot \text{s/m}^2$, respectively. An experiment is conducted in which a long aluminum rod of diameter $D = 20 \text{ mm}$ and initial temperature $T_i = 32^\circ\text{C}$ is suddenly immersed horizontally into a large bath of the fluid at a temperature of $T_\infty = 22^\circ\text{C}$. At $t = 65 \text{ s}$, the measured temperature of the rod is $T_f = 23^\circ\text{C}$. Determine the thermal expansion coefficient of the fluid β .

SOLUTION

Known: Initial and final temperatures of aluminum rod of known diameter. Temperature and properties of the fluid.

Find: Thermal expansion coefficient of the fluid.

Schematic:



Assumptions:

1. Constant properties.
2. Spatially uniform rod temperature (applicability of the lumped capacitance approximation).

Properties: Table A.1, aluminum ($\bar{T} = 300 \text{ K}$): $\rho_s = 2702 \text{ kg/m}^3$, $c_{p,s} = 903 \text{ J/kg} \cdot \text{K}$, $k_s = 237 \text{ W/m} \cdot \text{K}$.

Analysis: Because the temperature difference between the rod and the fluid decreases with time, we expect the convection heat transfer coefficient to decrease as cooling

proceeds. Since $\bar{h}$ depends on buoyancy forces established by temperature differences, the analysis of Section 5.3.3 may be applied. From Equations 5.28 and 5.26

$$\frac{\theta}{\theta_i} = \left[\frac{n C_1 A_{s,c} \theta_i^n}{\rho_s V c_{p,s}} t + 1 \right]^{-1/n} \quad (1)$$

where $\bar{h} = C_1 (T_s - T_\infty)^n$ and $\theta = T_s - T_\infty$. From Equation 9.33, $\overline{Nu}_D = C Ra_D^n$. Substituting definitions of the Nusselt and Rayleigh numbers into Equation 9.33 yields

$$\bar{h} = C \frac{k_l}{D} \left[\frac{g \beta D^3}{\nu_l \alpha_l} \right]^n (T_s - T_\infty)^n \quad (2)$$

From a comparison of Equation 2 with the expression $\bar{h} = C_1 (T_s - T_\infty)^n$, it is evident that

$$C_1 = C \frac{k_l}{D} \left[\frac{g \beta D^3}{\nu_l \alpha_l} \right]^n \quad (3)$$

Defining a final excess temperature as $\theta_f = T_{s,f} - T_\infty$ at $t = t_f$ and noting that $\nu_l = \mu_l / \rho_l$, $\alpha_l = k_l / \rho_l c_{p,l}$, and $A_{s,c} / V = 4 / D$, Equation 3 may be substituted into Equation 1, yielding

$$\beta = \frac{k_l \mu_l}{c_{p,l} \rho_l^2 g D^3} \left\{ \frac{\rho_s c_{p,s} D^2}{4 k_l C n t_f \theta_i^n} \left[\left(\frac{\theta_f}{\theta_i} \right)^{-n} - 1 \right] \right\}^{1/n} \quad (4)$$

For now we will assume the Rayleigh number falls in the range $10^4 \leq Ra_D \leq 10^7$, for which $C = 0.480$ and $n = 0.250$ from Table 9.2. Hence the thermal expansion coefficient is

$$\begin{aligned} \beta &= \frac{0.705 \text{ W/m} \cdot \text{K} \times 962 \times 10^{-6} \text{ N} \cdot \text{s/m}^2}{3587 \text{ J/kg} \cdot \text{K} \times (1146 \text{ kg/m}^3)^2 \times 9.8 \text{ m/s}^2 \times (20 \times 10^{-3} \text{ m})^3} \\ &\times \left\{ \frac{2702 \text{ kg/m}^3 \times 903 \text{ J/kg} \cdot \text{K} \times (20 \times 10^{-3} \text{ m})^2}{4 \times 0.705 \text{ W/m} \cdot \text{K} \times 0.480 \times 0.25 \times 65 \text{ s} \times (10 \text{ K})^{0.25}} \left[\left(\frac{1.0}{10} \right)^{-0.25} - 1 \right] \right\}^{1/0.25} \\ &= 261 \times 10^{-6} \text{ K}^{-1} \end{aligned} \quad \triangleleft$$

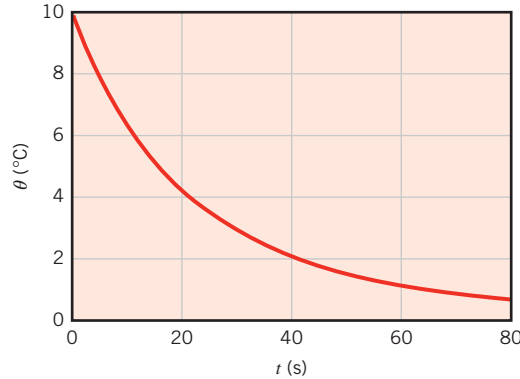
Using this value of the thermal expansion coefficient, the Rayleigh number based on the initial temperature difference is

$$\begin{aligned} Ra_{D,\max} &= \frac{g \beta \theta_i D^3}{\nu_l \alpha_l} = \frac{g \beta \rho^2 c_{p,l} \theta_i D^3}{\mu_l k_l} \\ &= \frac{9.8 \text{ m/s}^2 \times 261 \times 10^{-6} \text{ K}^{-1} \times (1146 \text{ kg/m}^3)^2 \times 3587 \text{ J/kg} \cdot \text{K} \times 10 \text{ K} \times (20 \times 10^{-3} \text{ m})^3}{962 \times 10^{-6} \text{ N} \cdot \text{s/m}^2 \times 0.705 \text{ W/m} \cdot \text{K}} \\ &= 1.42 \times 10^6 \end{aligned}$$

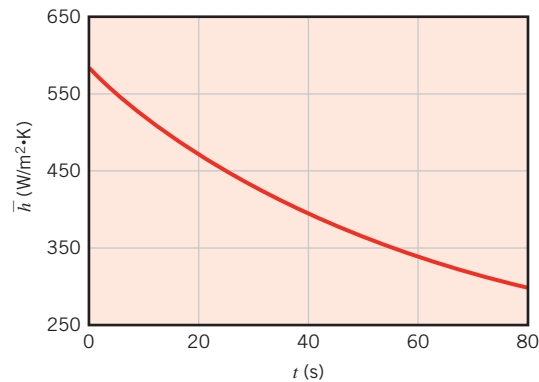
Since $\theta_f = \theta_i / 10$, the minimum value of the Rayleigh number during the cooling process is $Ra_{D,\min} = Ra_{D,\max} / 10 = 1.42 \times 10^5$. Therefore, $10^4 < Ra_{D,\min} < Ra_{D,\max} < 10^7$, and the values of C and n selected from Table 9.2 are appropriate. Hence the foregoing value of the thermal expansion coefficient is correct.

Comments:

1. The manner in which the rod temperature decreases during the cooling process may be determined from Equation 1 with C_1 obtained from Equation 3. The rod temperature history is shown in the figure.



2. Since the temperature difference between the rod and the fluid decreases with time, the Rayleigh number also decreases as cooling proceeds. This leads to a gradual reduction in the convection heat transfer coefficient during cooling, as can be determined by solving Equation 2 once $\theta(t)$ is known.



3. The maximum value of the convection heat transfer coefficient is $\bar{h}_{\max} = 584 \text{ W/m}^2 \cdot \text{K}$. This corresponds to a maximum Biot number of $Bi_{\max} = \bar{h}_{\max} (D/2) / k_s = 584 \text{ W/m}^2 \cdot \text{K} \times (20 \times 10^{-3} \text{ m} / 2) / 237 \text{ W/m} \cdot \text{K} = 0.025$ when the criterion of Equation 5.10 is applied in a conservative fashion. Since $Bi_{\max} < 0.1$, we conclude that the lumped capacitance approximation is valid.
4. Because the rod temperature continually decreases, the buoyancy forces within the fluid decrease with time. Hence fluid velocities continually evolve as the temperature difference between the rod and the fluid slowly decays. Equation 9.33 is strictly applicable only for steady-state conditions. In applying the correlation here, we have implicitly assumed that the instantaneous heat transfer rate from the rod is the same as the steady-state heat transfer rate *if* the same temperature difference exists between the rod and the fluid. This assumption often yields predictions of acceptable accuracy and is referred to as the *quasi-steady approximation*.

9.6.4 Spheres

The following correlation due to Churchill [10] is recommended for spheres in fluids of $Pr \gtrsim 0.7$ and for $Ra_D \lesssim 10^{11}$.

$$\overline{Nu}_D = 2 + \frac{0.589 Ra_D^{1/4}}{[1 + (0.469/Pr)^{9/16}]^{4/9}} \quad (9.35)$$

In the limit as $Ra_D \rightarrow 0$, Equation 9.35 reduces to $\overline{Nu}_D = 2$, which corresponds to heat transfer by conduction between a spherical surface and a stationary infinite medium, in a manner consistent with Equations 7.56 and 7.57.

Recommended correlations from this section are summarized in Table 9.3. Results for other immersed geometries and special conditions are presented in comprehensive reviews by Churchill [10] and Raithby and Hollands [21].

TABLE 9.3 Summary of free convection empirical correlations for immersed geometries

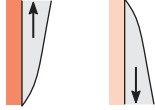
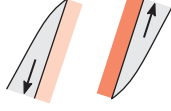
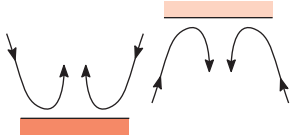
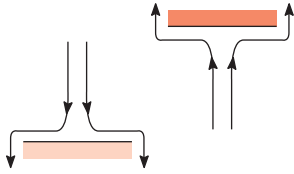
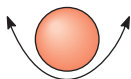
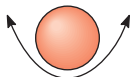
Geometry	Recommended Correlation	Restrictions
1. Vertical plates ^a 	Equation 9.26	None
2. Inclined plates Cold surface up or hot surface down 	Equation 9.26 $g \rightarrow g \cos \theta$	$0 \leq \theta \lesssim 60^\circ$
3. Horizontal plates (a) Hot surface up or cold surface down 	Equation 9.30 Equation 9.31	$10^4 \lesssim Ra_L \lesssim 10^7, Pr \gtrsim 0.7$ $10^7 \lesssim Ra_L \lesssim 10^{11}$
(b) Cold surface up or hot surface down 	Equation 9.32	$10^4 \lesssim Ra_L \lesssim 10^9, Pr \gtrsim 0.7$

TABLE 9.3 Continued

Geometry	Recommended Correlation	Restrictions
4. Horizontal cylinder 	Equation 9.34	$Ra_D \lesssim 10^{12}$
5. Sphere 	Equation 9.35	$Ra_D \lesssim 10^{11}$ $Pr \gtrsim 0.7$

^aThe correlation may be applied to a vertical cylinder if $(D/L) \gtrsim (35/Gr_L^{1/4})$.

9.7 Free Convection Within Parallel Plate Channels

A common free convection geometry involves vertical (or inclined) parallel plate channels that are open to the ambient at opposite ends (Figure 9.9). The plates could constitute a fin array used to enhance free convection heat transfer from a base surface to which the fins are attached, or they could constitute an array of circuit boards with heat-dissipating electronic components. Surface thermal conditions may be idealized as being isothermal or isoflux and symmetrical ($T_{s,1} = T_{s,2}$; $q''_{s,1} = q''_{s,2}$) or asymmetrical ($T_{s,1} \neq T_{s,2}$; $q''_{s,1} \neq q''_{s,2}$).

For vertical channels ($\theta = 0$) buoyancy acts exclusively to induce motion in the streamwise (x) direction and, beginning at $x = 0$, boundary layers develop on each surface. For short channels and/or large spacings (small L/S), independent boundary layer development occurs at each surface and conditions correspond to those for an *isolated plate* in an infinite, quiescent medium. For large L/S , however, boundary layers developing on opposing surfaces eventually merge to yield a fully developed condition. If the channel is inclined, there is a component of the buoyancy force normal, as well as parallel, to the streamwise direction, and conditions may be strongly influenced by development of a three-dimensional, secondary flow.

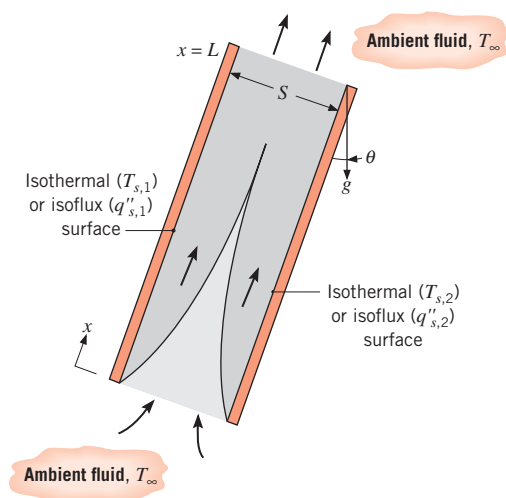


FIGURE 9.9 Free convection flow between hot parallel plates with opposite ends exposed to a quiescent fluid.

9.7.1 Vertical Channels

Beginning with the benchmark paper by Elenbaas [24], the vertical orientation has been studied extensively for plates with symmetric and asymmetric isothermal or isoflux surface conditions. For *isothermal plates of the same temperature*, Elenbaas obtained the following semiempirical correlation:

$$\overline{Nu}_S = \frac{1}{24} Ra_S \left(\frac{S}{L} \right) \left\{ 1 - \exp \left[- \frac{35}{Ra_S (S/L)} \right] \right\}^{3/4} \quad (9.36)$$

where the average Nusselt and Rayleigh numbers are defined as

$$\overline{Nu}_S = \left(\frac{q/A}{T_s - T_\infty} \right) \frac{S}{k} \quad (9.37)$$

and

$$Ra_S = \frac{g\beta(T_s - T_\infty)S^3}{\nu\alpha} \quad (9.38)$$

Equation 9.36 was developed for air as the working fluid, and its range of applicability is

$$\left[10^{-1} \lesssim \frac{S}{L} Ra_S \lesssim 10^5 \right]$$

Knowledge of the average Nusselt number for a plate therefore permits determination of the total heat rate for the plate. In the fully developed limit ($S/L \rightarrow 0$), Equation 9.36 reduces to

$$\overline{Nu}_{S(\text{fd})} = \frac{Ra_S (S/L)}{24} \quad (9.39)$$

Retention of the L dependence results from defining $\overline{Nu}_S$ in terms of the fixed inlet (ambient) temperature and not in terms of the fluid mixed-mean temperature, which is not explicitly known. For the common condition corresponding to adjoining isothermal ($T_{s,1}$) and insulated ($q''_{s,2} = 0$) plates, the fully developed limit yields the following expression for the isothermal surface [25]:

$$\overline{Nu}_{S(\text{fd})} = \frac{Ra_S (S/L)}{12} \quad (9.40)$$

For isoflux surfaces, it is more convenient to define a local Nusselt number as

$$Nu_{S,L} = \left(\frac{q''_s}{T_{s,L} - T_\infty} \right) \frac{S}{k} \quad (9.41)$$

and to correlate results in terms of a modified Rayleigh number defined as

$$Ra_S^* = \frac{g\beta q''_s S^4}{k\nu\alpha} \quad (9.42)$$

The subscript L refers to conditions at $x = L$, where the plate temperature is a maximum. For symmetric, isoflux plates the fully developed limit corresponds to [25]

$$Nu_{S,L(\text{fd})} = 0.144 [Ra_S^*(S/L)]^{1/2} \quad (9.43)$$

and for asymmetric isoflux conditions with one surface insulated ($q''_{s,2} = 0$) the limit is

$$Nu_{S,L(\text{fd})} = 0.204 [Ra_S^*(S/L)]^{1/2} \quad (9.44)$$

Combining the foregoing relations for the fully developed limit with available results for the isolated plate limit, Bar-Cohen and Rohsenow [25] obtained Nusselt number correlations applicable to the complete range of S/L . For isothermal and isoflux conditions, respectively, the correlations are of the form

$$\overline{Nu}_S = \left[\frac{C_1}{(Ra_S S/L)^2} + \frac{C_2}{(Ra_S S/L)^{1/2}} \right]^{-1/2} \quad (9.45)$$

$$Nu_{S,L} = \left[\frac{C_1}{Ra_S^* S/L} + \frac{C_2}{(Ra_S^* S/L)^{2/5}} \right]^{-1/2} \quad (9.46)$$

where the constants C_1 and C_2 are given in Table 9.4 for the different surface thermal conditions. In each case the fully developed and isolated plate limits correspond to Ra_S (or Ra_S^*)/ $S/L \lesssim 10$ and Ra_S (or Ra_S^*)/ $S/L \gtrsim 100$, respectively.

Bar-Cohen and Rohsenow [25] used the foregoing correlations to infer the optimum plate spacing S_{opt} for maximizing heat transfer from an array of isothermal plates, as well as the spacing S_{max} needed to maximize heat transfer from each plate in the array. Existence of an optimum for the array results from the fact that, although heat transfer from each plate decreases with decreasing S , the number of plates that may be placed in a prescribed volume increases. Hence S_{opt} maximizes heat transfer from the array by yielding a maximum for the product of $\bar{h}$ and the total plate surface area. In contrast, to maximize heat transfer from each plate, S_{max} must be large enough to preclude overlap of adjoining boundary layers, such that the isolated plate limit remains valid over the entire plate.

Consideration of the optimum plate spacing is particularly important for vertical parallel plates used as fins to enhance heat transfer by natural convection from a base surface of fixed width W . With the temperature of the fins exceeding that of the ambient fluid, flow between the fins is induced by buoyancy forces. However, resistance to the flow is associated with viscous forces imposed by the surface of the fins, and the rate of mass flow between adjoining fins is governed by a balance between buoyancy and viscous forces. Since viscous forces increase with decreasing S , there is an accompanying reduction in the

TABLE 9.4 Heat transfer parameters for free convection
between vertical parallel plates

Surface Condition	C_1	C_2	S_{opt}	$S_{\text{max}}/S_{\text{opt}}$
Symmetric isothermal plates ($T_{s,1} = T_{s,2}$)	576	2.87	$2.71(Ra_S/S^3L)^{-1/4}$	1.71
Symmetric isoflux plates ($q''_{s,1} = q''_{s,2}$)	48	2.51	$2.12(Ra_S^*/S^4L)^{-1/5}$	4.77
Isothermal/adiabatic plates ($T_{s,1}, q''_{s,2} = 0$)	144	2.87	$2.15(Ra_S/S^3L)^{-1/4}$	1.71
Isoflux/adiabatic plates ($q''_{s,1}, q''_{s,2} = 0$)	24	2.51	$1.69(Ra_S^*/S^4L)^{-1/5}$	4.77

flow rate, and hence $\bar{h}$. However, for fixed W , the attendant increase in the number of fins increases the total surface area A_s and yields a maximum in $\bar{h}A_s$ for $S = S_{\text{opt}}$. For $S < S_{\text{opt}}$, the amount by which $\bar{h}$ is diminished by viscous effects exceeds the increase in A_s ; for $S > S_{\text{opt}}$, the amount by which A_s is diminished exceeds the increase in $\bar{h}$.

For isoflux plates, the total volumetric heat rate simply increases with decreasing S . However, the need to maintain T_s below prescribed limits precludes reducing S to extremely small values. Hence S_{opt} may be defined as that value of S which yields the maximum volumetric heat dissipation per unit temperature difference, $T_s(L) - T_\infty$. The spacing S_{max} that yields the lowest possible surface temperature for a prescribed heat flux, without regard to volumetric considerations, is again the value of S that precludes boundary layer merger. Values of S_{opt} and $S_{\text{max}}/S_{\text{opt}}$ are presented in Table 9.4 for plates of negligible thickness.

In using the foregoing correlations, fluid properties are evaluated at average temperatures of $\bar{T} = (T_s + T_\infty)/2$ for isothermal surfaces and $\bar{T} = (T_{s,L} + T_\infty)/2$ for isoflux surfaces.

9.7.2 Inclined Channels

Experiments have been performed by Azevedo and Sparrow [16] for inclined channels in water. Symmetric isothermal plates and isothermal-insulated plates were considered for $0 \leq \theta \leq 45^\circ$ and conditions within the isolated plate limit, $Ra_s(S/L) > 200$. Although three-dimensional secondary flows were observed at the lower plate, when it was hot, data for all experimental conditions were correlated to within $\pm 10\%$ by

$$\overline{Nu}_s = 0.645[Ra_s(S/L)]^{1/4} \quad (9.47)$$

Departures of the data from the correlation were most pronounced at large tilt angles with bottom surface heating and were attributed to heat transfer enhancement by the three-dimensional secondary flow. Fluid properties are evaluated at $\bar{T} = (T_s + T_\infty)/2$.

9.8 Empirical Correlations: Enclosures

The foregoing results pertain to free convection between a surface and an extensive fluid medium. However, engineering applications frequently involve heat transfer between surfaces that are at different temperatures and are separated by an *enclosed* fluid. In this section we present correlations that are pertinent to several common geometries.

9.8.1 Rectangular Cavities

The rectangular cavity (Figure 9.10) has been studied extensively, and comprehensive reviews of both experimental and theoretical results are available [26, 27]. Two of the opposing walls are maintained at different temperatures ($T_1 > T_2$), while the remaining walls are insulated from the surroundings. The tilt angle τ between the hot and cold surfaces

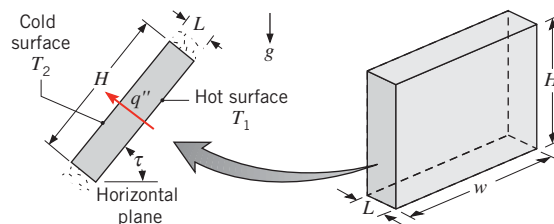


FIGURE 9.10 Free convection in a rectangular cavity.

and the horizontal can vary from 0° (*horizontal cavity* with bottom surface hot) to 90° (*vertical cavity* with hot and cold sidewalls) to 180° (*horizontal cavity* with hot top surface). The heat flux across the cavity, which is expressed as

$$q'' = h(T_1 - T_2) \quad (9.48)$$

can depend strongly on the aspect ratio H/L , as well as the value of τ . For large values of the aspect ratio w/L , its dependence on w/L is small and may be neglected for the purposes of this text.

The horizontal cavity heated from below ($\tau = 0$) has been considered by many investigators. For H/L , $w/L \gg 1$, and Rayleigh numbers less than a critical value of $Ra_{L,c} = 1708$, buoyancy forces cannot overcome the resistance imposed by viscous forces and there is no advection within the cavity. Hence heat transfer from the bottom to the top surface occurs by conduction or, for a gas, by conduction and radiation. Since conditions correspond to one-dimensional conduction through a plane fluid layer, the convection coefficient is $h = k/L$ and $Nu_L = 1$. However, for

$$Ra_L \equiv \frac{g\beta(T_1 - T_2)L^3}{\nu\alpha} > 1708$$

conditions are thermally unstable and there is advection within the cavity. For Rayleigh numbers in the range $1708 < Ra_L \lesssim 5 \times 10^4$, fluid motion consists of regularly spaced roll cells (Figure 9.11), while for larger Rayleigh numbers, the cells break down and the fluid motion evolves through many different patterns before becoming turbulent.

As a first approximation, convection coefficients for the horizontal cavity heated from below may be obtained from the following correlation proposed by Globe and Dropkin [28]:

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = 0.069 Ra_L^{1/3} Pr^{0.074} \quad 3 \times 10^5 \lesssim Ra_L \lesssim 7 \times 10^9 \quad (9.49)$$

where all properties are evaluated at the average temperature, $\bar{T} \equiv (T_1 + T_2)/2$. The correlation applies for values of L/H sufficiently small to ensure a negligible effect of the sidewalls. More detailed correlations, which apply over a wider range of Ra_L , have been proposed [29, 30]. In concluding the discussion of horizontal cavities, it is noted that in the absence of radiation, for heating from above ($\tau = 180^\circ$), heat transfer from the top to the bottom surface is exclusively by conduction ($Nu_L = 1$), irrespective of the value of Ra_L .

In the vertical rectangular cavity ($\tau = 90^\circ$), the vertical surfaces are hot and cold, while the horizontal surfaces are adiabatic. As shown in Figure 9.12, fluid motion is characterized by a recirculating or cellular flow for which fluid ascends along the hot wall and descends along the cold wall. For small Rayleigh numbers, $Ra_L \lesssim 10^3$, the buoyancy-driven flow is weak and, in the absence of radiation, heat transfer is primarily by conduction across the fluid. Hence, from Fourier's law, the Nusselt number is again $Nu_L = 1$. With increasing Rayleigh number,

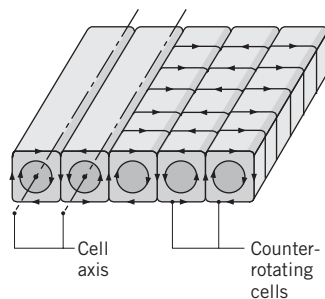


FIGURE 9.11 Longitudinal roll cells characteristic of advection in a horizontal fluid layer heated from below ($1708 < Ra_L \lesssim 5 \times 10^4$).

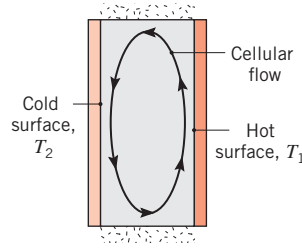


FIGURE 9.12 Cellular flow in a vertical cavity with different sidewall temperatures.

the cellular flow intensifies and becomes concentrated in thin boundary layers adjoining the sidewalls. The core becomes nearly stagnant, although additional cells can develop in the corners and the sidewall boundary layers eventually undergo transition to turbulence. For aspect ratios in the range $1 \lesssim (H/L) \lesssim 10$, the following correlations have been suggested [27]:

$$\overline{Nu}_L = 0.22 \left(\frac{Pr}{0.2 + Pr} Ra_L \right)^{0.28} \left(\frac{H}{L} \right)^{-1/4} \quad (9.50)$$

$$\left[\begin{array}{l} 2 \lesssim \frac{H}{L} \lesssim 10 \\ Pr \lesssim 10^5 \\ 10^3 \lesssim Ra_L \lesssim 10^{10} \end{array} \right]$$

$$\overline{Nu}_L = 0.18 \left(\frac{Pr}{0.2 + Pr} Ra_L \right)^{0.29} \quad (9.51)$$

$$\left[\begin{array}{l} 1 \lesssim \frac{H}{L} \lesssim 2 \\ 10^{-3} \lesssim Pr \lesssim 10^5 \\ 10^3 \lesssim \frac{Ra_L Pr}{0.2 + Pr} \end{array} \right]$$

while for larger aspect ratios, the following correlations have been proposed [31]:

$$\overline{Nu}_L = 0.42 Ra_L^{1/4} Pr^{0.012} \left(\frac{H}{L} \right)^{-0.3} \left[\begin{array}{l} 10 \lesssim \frac{H}{L} \lesssim 40 \\ 1 \lesssim Pr \lesssim 2 \times 10^4 \\ 10^4 \lesssim Ra_L \lesssim 10^7 \end{array} \right] \quad (9.52)$$

$$\overline{Nu}_L = 0.046 Ra_L^{1/3} \left[\begin{array}{l} 1 \lesssim \frac{H}{L} \lesssim 40 \\ 1 \lesssim Pr \lesssim 20 \\ 10^6 \lesssim Ra_L \lesssim 10^9 \end{array} \right] \quad (9.53)$$

TABLE 9.5 Critical angle for inclined rectangular cavities

(H/L)	1	3	6	12	>12
τ^*	25°	53°	60°	67°	70°

Convection coefficients computed from the foregoing expressions are to be used with Equation 9.48. Again, all properties are evaluated at the mean temperature, $(T_1 + T_2)/2$.

Studies of free convection in tilted cavities are often stimulated by applications involving flat-plate solar collectors [32–37]. For such cavities, the fluid motion consists of a combination of the roll structure of Figure 9.11 and the cellular structure of Figure 9.12. Typically, transition between the two types of fluid motion occurs at a critical tilt angle, τ^* , with a corresponding change in the value of $\overline{Nu}_L$. For large aspect ratios, $(H/L) \gtrsim 12$, and tilt angles less than the critical value τ^* given in Table 9.5, the following correlation due to Hollands et al. [37] is in excellent agreement with available data:

$$\begin{aligned} \overline{Nu}_L = 1 + 1.44 \left[1 - \frac{1708}{Ra_L \cos \tau} \right] \cdot \left[1 - \frac{1708 (\sin 1.8\tau)^{1.6}}{Ra_L \cos \tau} \right] \\ + \left[\left(\frac{Ra_L \cos \tau}{5830} \right)^{1/3} - 1 \right] \cdot \left[\begin{array}{l} \frac{H}{L} \gtrsim 12 \\ 0 < \tau \lesssim \tau^* \end{array} \right] \end{aligned} \quad (9.54)$$

The notation $[\]$ implies that, if the quantity in brackets is negative, it must be set equal to zero. The implication is that, if the Rayleigh number is less than a critical value $Ra_{L,c} = 1708/\cos \tau$, there is no flow within the cavity. For small aspect ratios Catton [27] suggests that reasonable results may be obtained from a correlation of the form

$$\overline{Nu}_L = \overline{Nu}_L(\tau = 0) \left[\frac{\overline{Nu}_L(\tau = 90^\circ)}{\overline{Nu}_L(\tau = 0)} \right]^{\tau/\tau^*} (\sin \tau^*)^{(\tau/4\tau^*)} \left[\begin{array}{l} \frac{H}{L} \lesssim 12 \\ 0 < \tau \lesssim \tau^* \end{array} \right] \quad (9.55)$$

Beyond the critical tilt angle, the following correlations due to Ayyaswamy and Catton [32] and Arnold et al. [35], respectively, have been recommended [27] for all aspect ratios (H/L):

$$\overline{Nu}_L = \overline{Nu}_L(\tau = 90^\circ) (\sin \tau)^{1/4} \quad \tau^* \lesssim \tau < 90^\circ \quad (9.56)$$

$$\overline{Nu}_L = 1 + [\overline{Nu}_L(\tau = 90^\circ) - 1] \sin \tau \quad 90^\circ < \tau < 180^\circ \quad (9.57)$$

9.8.2 Concentric Cylinders

Free convection heat transfer in the annular space between *long*, horizontal concentric cylinders (Figure 9.13) has been considered by Raithby and Hollands [38]. Flow in the annular region is characterized by two cells that are symmetric about the vertical mid-plane. If the inner cylinder is hot and the outer cylinder is cold ($T_i > T_o$), fluid ascends and descends along the inner and outer cylinders, respectively. If $T_i < T_o$, the cellular flows are

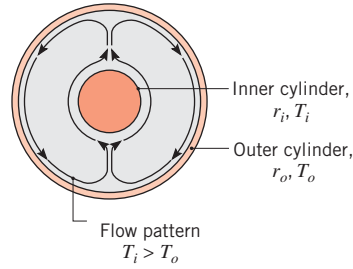


FIGURE 9.13 Free convection flow in the annular space between long, horizontal, concentric cylinders or concentric spheres of inner radius r_i and outer radius r_o .

reversed. The convection heat transfer rate (W) between the two cylinders, each of length L , is expressed by Equation 3.32 (with an *effective thermal conductivity*, k_{eff} , replacing the molecular thermal conductivity, k) as

$$q = \frac{2\pi L k_{\text{eff}} (T_i - T_o)}{\ln(r_o/r_i)} \quad (9.58)$$

We see that the effective conductivity of a fictitious *stationary* fluid will transfer the same amount of heat as the actual *moving* fluid. The suggested correlation for k_{eff} is

$$\frac{k_{\text{eff}}}{k} = 0.386 \left(\frac{Pr}{0.861 + Pr} \right)^{1/4} Ra_c^{1/4} \quad (9.59)$$

where the length scale in Ra_c is given by

$$L_c = \frac{2[\ln(r_o/r_i)]^{4/3}}{(r_i^{-3/5} + r_o^{-3/5})^{5/3}} \quad (9.60)$$

Equation 9.59 may be used for the range $0.7 \lesssim Pr \lesssim 6000$ and $Ra_c \lesssim 10^7$. Properties are evaluated at the mean temperature, $T_m = (T_i + T_o)/2$. Of course, the minimum heat transfer rate between the cylinders cannot fall below the conduction limit; therefore, $k_{\text{eff}} = k$ if the value of k_{eff}/k predicted by Equation 9.59 is less than unity. A more detailed correlation, which accounts for cylinder eccentricity effects, has been developed by Kuehn and Goldstein [39].

9.8.3 Concentric Spheres

Raithby and Hollands [38] have also considered free convection heat transfer between concentric spheres (Figure 9.13) and express the convection heat transfer rate by Equation 3.40 (with an effective thermal conductivity, k_{eff} , replacing the molecular thermal conductivity, k) as

$$q = \frac{4\pi k_{\text{eff}} (T_i - T_o)}{(1/r_i) - (1/r_o)} \quad (9.61)$$

The effective thermal conductivity is

$$\frac{k_{\text{eff}}}{k} = 0.74 \left(\frac{Pr}{0.861 + Pr} \right)^{1/4} Ra_s^{1/4} \quad (9.62)$$

where the length scale in Ra_s is given by

$$L_s = \frac{\left(\frac{1}{r_i} - \frac{1}{r_o}\right)^{4/3}}{2^{1/3}(r_i^{-7/5} + r_o^{-7/5})^{5/3}} \quad (9.63)$$

The result may be used to a reasonable approximation for $0.7 \lesssim Pr \lesssim 4000$ and $Ra_s \lesssim 10^4$. Properties are evaluated at $T_m = (T_i + T_o)/2$, and $k_{\text{eff}} = k$ if the value of k_{eff}/k predicted by Equation 9.62 is less than unity.

EXAMPLE 9.5

A long tube of 0.1-m diameter is maintained at 120°C by passing steam through its interior. A radiation shield is installed concentric to the tube with an air gap of 10 mm. If the shield is at 35°C, estimate the rate of heat transfer by free convection from the tube per unit length. What is the rate of heat loss if the space between the tube and the shield is filled with glass-fiber blanket insulation?

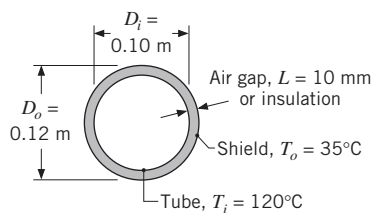
SOLUTION

Known: Temperatures and diameters of a steam tube and a concentric radiation shield.

Find:

1. Rate of heat loss per unit length of tube.
2. Rate of heat loss if air space is filled with glass-fiber blanket insulation.

Schematic:



Assumptions:

1. Radiation heat transfer may be neglected.
2. Contact resistance with insulation is negligible.
3. Ideal gas.
4. Constant properties.

Properties: Table A.4, air [$T = (T_i + T_o)/2 = 350$ K]: $k = 0.030$ W/m · K, $\nu = 20.92 \times 10^{-6}$ m²/s, $\alpha = 29.9 \times 10^{-6}$ m²/s, $Pr = 0.70$, $\beta = 0.00285$ K⁻¹. Table A.3, insulation, glass-fiber ($T \approx 300$ K): $k = 0.038$ W/m · K.

Analysis:

1. From Equation 9.58, the rate of heat loss per unit length by free convection is

$$q' = \frac{2\pi k_{\text{eff}}(T_i - T_o)}{\ln(r_o/r_i)}$$

where k_{eff} may be obtained from Equations 9.59 and 9.60. With

$$L_c = \frac{2[\ln(r_o/r_i)]^{4/3}}{(r_i^{-3/5} + r_o^{-3/5})^{5/3}} = \frac{2[\ln(0.06 \text{ m}/0.05 \text{ m})]^{4/3}}{(0.05^{-3/5} + 0.06^{-3/5})^{5/3} \text{ m}^{-1}} = 0.00117 \text{ m}$$

we find

$$Ra_c = \frac{g\beta(T_i - T_o)L_c^3}{\nu\alpha} = \frac{9.8 \text{ m/s}^2 \times 0.00285 \text{ K}^{-1} \times (120 - 35)^\circ\text{C} \times (0.00117 \text{ m})^3}{20.92 \times 10^{-6} \text{ m}^2/\text{s} \times 29.9 \times 10^{-6} \text{ m}^2/\text{s}} = 171$$

The effective thermal conductivity is then

$$\begin{aligned} k_{\text{eff}} &= 0.386k \left(\frac{Pr}{0.861 + Pr} \right)^{1/4} Ra_c^{1/4} \\ &= 0.386 \times 0.030 \text{ W/m} \cdot \text{K} \left(\frac{0.70}{0.861 + 0.70} \right)^{1/4} (171)^{1/4} = 0.0343 \text{ W/m} \cdot \text{K} \end{aligned}$$

and the rate of heat loss is

$$q' = \frac{2\pi k_{\text{eff}}(T_i - T_o)}{\ln(r_o/r_i)} = \frac{2\pi(0.0343 \text{ W/m} \cdot \text{K})}{\ln(0.06 \text{ m}/0.05 \text{ m})} (120 - 35)^\circ\text{C} = 100 \text{ W/m} \quad \triangleleft$$

2. With insulation in the space between the tube and the shield, heat loss is by conduction; comparing Equation 3.32 and Equation 9.58,

$$q'_{\text{ins}} = q' \frac{k_{\text{ins}}}{k_{\text{eff}}} = 100 \text{ W/m} \frac{0.038 \text{ W/m} \cdot \text{K}}{0.0343 \text{ W/m} \cdot \text{K}} = 111 \text{ W/m} \quad \triangleleft$$

Comments: Although there is slightly more heat loss by conduction through the insulation than by free convection across the air space, the total heat loss across the air space may exceed that through the insulation because of the effects of radiation. The heat loss due to radiation may be minimized by using a radiation shield of low emissivity, and the means for calculating the loss will be developed in Chapter 13.

9.9 Combined Free and Forced Convection

In dealing with forced convection (Chapters 6 through 8), we ignored the effects of free convection. This was, of course, an assumption; for, as we now know, buoyancy forces develop in fluids as a result of temperature gradients, and unstable gradients might induce free convection. Similarly, in the preceding sections of this chapter, we assumed

that forced convection was negligible. It is now time to acknowledge that situations may arise for which free and forced convection effects are comparable, in which case it is inappropriate to neglect either process. In Section 9.3 we indicated that free convection is negligible if $(Gr_L/Re_L^2) \ll 1$ and that forced convection is negligible if $(Gr_L/Re_L^2) \gg 1$. Hence the *combined free and forced* (or *mixed*) *convection* regime is generally one for which $(Gr_L/Re_L^2) \approx 1$.

The effect of buoyancy on heat transfer in a forced flow is strongly influenced by the direction of the buoyancy force relative to that of the flow. Three special cases that have been studied extensively correspond to buoyancy-induced and forced motions having the same direction (*assisting* flow), opposite directions (*opposing* flow), and perpendicular directions (*transverse* flow). Upward and downward forced motions over a hot vertical plate are examples of assisting and opposing flows, respectively. Examples of transverse flow include horizontal motion over a hot cylinder, sphere, or horizontal plate. In assisting and transverse flows, buoyancy acts to enhance the rate of heat transfer associated with pure forced convection; in opposing flows, it acts to decrease this rate.

It has become common practice to correlate mixed convection heat transfer results for external and internal flows by an expression of the form

$$Nu^n = Nu_F^n \pm Nu_N^n \quad (9.64)$$

For the specific geometry of interest, the Nusselt numbers Nu_F and Nu_N are determined from correlations for pure forced and natural (free) convection, respectively. The plus sign on the right-hand side of Equation 9.64 applies for assisting and transverse flows, while the minus sign applies for opposing flow. The best correlation of data is often obtained for $n = 3$, although values of 7/2 and 4 may be better suited for transverse flows involving horizontal plates and cylinders (or spheres), respectively.

Equation 9.64 should be viewed as a first approximation, and any serious treatment of a mixed convection problem should be accompanied by an examination of the open literature. Mixed convection flows received considerable attention in the late 1970s to the 1990s, and comprehensive literature reviews are available [40–43]. The flows are endowed with a variety of rich and unusual features that can complicate heat transfer predictions. For example, in a horizontal, parallel-plate channel, three-dimensional flows in the form of longitudinal vortices are induced by bottom heating, and the longitudinal variation of the Nusselt number is characterized by a decaying oscillation [44, 45]. Moreover, in channel flows, significant asymmetries may be associated with convection heat transfer at top and bottom surfaces [46]. Finally, we note that, although buoyancy effects can significantly enhance heat transfer for laminar forced convection flows, enhancement is typically negligible if the forced flow is turbulent [47].

9.10 Convection Mass Transfer

The buoyancy term on the right-hand side of Equation 9.2 is due to density variations in the fluid, which may arise from species concentration gradients, as well as temperature gradients. Hence, a more general form of the Grashof number, Equation 9.10, is

$$Gr_L = \frac{g(\Delta\rho/\rho)L^3}{\nu^2} = \frac{g(\rho_s - \rho_\infty)L^3}{\rho\nu^2} \quad (9.65)$$

which may be applied to natural convection flows driven by concentration gradients and/or temperature gradients. As shown in Section 9.2, if density variations are due only to temperature gradients, $(\Delta\rho/\rho) = -\beta\Delta T$. However, if there are no temperature gradients, motion may still be induced by spatial variations in the species composition, and similarity considerations lead to the conclusion that $Sh_L = f(Gr_L, Sc)$. Moreover, correlations for convection mass transfer may be inferred from those for heat transfer by invoking the heat and mass transfer analogy. For example, if species A is evaporating or sublimating from a vertical surface into a quiescent ambient fluid B, the convection mass transfer coefficient may be obtained from the analogous form of Equation 9.24. That is,

$$\overline{Sh}_L = \frac{\overline{h}_m L}{D_{AB}} = C(Gr_L Sc)^n \quad (9.66)$$

where Gr_L is given by Equation 9.65. If the molecular weight of species A is less than that of species B, $\rho_s < \rho_\infty$ and the buoyancy-induced flow is upward along the surface. If the opposite is true, $\rho_s > \rho_\infty$ and the flow is descending.

The analogy may only be applied in the foregoing manner for isothermal conditions. If there are gradients in *both* temperature and species concentration, heat and mass transfer will occur concurrently by natural convection. Similarity considerations then yield $\overline{Nu}_L = f(Gr_L, Pr, Sc)$ and $\overline{Sh}_L = f(Gr_L, Sc, Pr)$, where the density difference $\Delta\rho$ is due to both temperature and concentration variations. As a first approximation, existing correlations of the form $\overline{Nu}_L = f(Gr_L, Pr)$ and $\overline{Sh}_L = f(Gr_L, Sc)$ may be used to determine the convection transfer coefficients, so long as the value of $\Delta\rho = \rho_s - \rho_\infty$ is calculated by including the effects of both temperature and concentration variations on ρ_s and ρ_∞ and $Le = Pr/Sc \approx 1$. In a binary mixture of species A and B, the surface and free stream densities are defined as $\rho_s = \rho_{s,A} + \rho_{s,B}$ and $\rho_\infty = \rho_{\infty,A} + \rho_{\infty,B}$, respectively, where the species densities depend on the surface and free stream temperatures. The average density across the boundary layer(s) is $\rho = (\rho_s + \rho_\infty)/2$. For cases involving fluids for which Le is much less or much greater than unity, the literature should be consulted.

9.11 Summary

We have considered convective flows that originate in part or exclusively from buoyancy forces, and we have introduced the dimensionless parameters needed to characterize such flows. You should be able to discern when free convection effects are important and to quantify the associated heat transfer rates. An assortment of empirical correlations has been provided for this purpose.

To test your understanding of related concepts, consider the following questions.

- What is an *extensive, quiescent* fluid?
- What conditions are required for a buoyancy-driven flow?
- How does the velocity profile in the free convection boundary layer on a hot vertical plate differ from the velocity profile in the boundary layer associated with forced, parallel flow over a flat plate?
- What is the general form of the buoyancy term in the x -momentum equation for a free convection boundary layer? How may it be approximated if the flow is due to temperature variations? What is the name of the approximation?

- What is the physical interpretation of the *Grashof number*? What is the *Rayleigh number*? How does each parameter depend on the characteristic length?
- For a hot horizontal plate in quiescent air, do you expect heat transfer to be larger for the top or bottom surface? Why? For a cold horizontal plate in quiescent air, do you expect heat transfer to be larger for the top or bottom surface? Why?
- For free convection within a vertical parallel plate channel, what kind of force balance governs the flow rate in the channel?
- For a vertical channel with isothermal plates, what is the physical basis for existence of an optimum spacing?
- What is the nature of flow in a cavity whose vertical surfaces are hot and cold? What is the nature of flow in an annular space between concentric cylindrical surfaces that are hot and cold?
- What is meant by the term *mixed convection*? How can one determine if mixed convection effects should be considered in a heat transfer analysis? Under what conditions is heat transfer enhanced by mixed convection? Under what conditions is it reduced?
- Consider transport of species A from a horizontal surface facing upward in a quiescent fluid B. If $T_s = T_\infty$ and the molecular weight of A is less than that of B, what is the analogous heat transfer problem? What is the analogous heat transfer problem if the molecular weight of A exceeds that of B?

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Problems

Properties and General Considerations

- 9.1** The one-dimensional plane wall of Figure 3.1 is of thickness $L = 75$ mm and thermal conductivity $k = 2.5$ W/m · K. The fluid temperatures are $T_{\infty,1} = 200^\circ\text{C}$

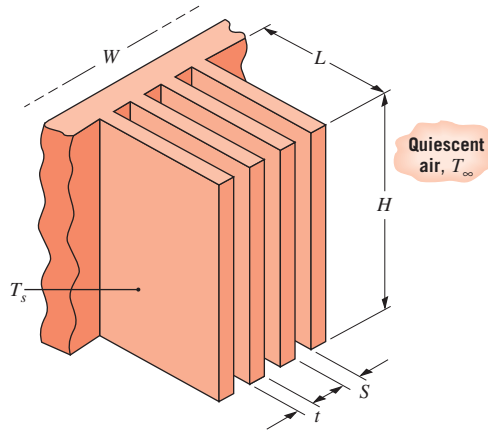
and $T_{\infty,2} = 100^\circ\text{C}$, respectively. Using the minimum and maximum typical values of the convection heat transfer coefficients listed in Table 1.1, determine the minimum and maximum steady-state heat fluxes through the wall for (i) free convection in gases, (ii) free convection

in liquids, (iii) forced convection in gases, (iv) forced convection in liquids, and (v) convection with phase change.

- 9.2** Using the values of density for water in Table A.6, calculate the volumetric thermal expansion coefficient at 300 K from its definition, Equation 9.3, and compare your result with the tabulated value.
- 9.3** An object of characteristic length $L = 0.5$ m is 10°C warmer than the surrounding fluid, which is flowing at a velocity of 0.5 m/s. Graph the Reynolds number, the Grashof number, and the ratio Gr_L/Re_L^2 over the range $10 \leq T_f \leq 90^\circ\text{C}$, where T_f is the film temperature. Consider the fluid to be either air at atmospheric pressure or liquid water. Use of *IHT* is recommended.
- 9.4** Consider an object of characteristic length 0.015 m and a situation for which the temperature difference is 10°C . Evaluating thermophysical properties at the prescribed conditions, determine the Rayleigh number for the following fluids: air (1 atm, 400 K), helium (1 atm, 400 K), glycerin (285 K), and water (310 K).
- 9.5** To assess the efficacy of different liquids for cooling by natural convection, it is convenient to introduce a *figure of merit*, F_N , which combines the influence of all pertinent fluid properties on the convection coefficient. If the Nusselt number is governed by an expression of the form, $Nu_L \sim Ra^n$, obtain the corresponding relationship between F_N and the fluid properties. For a representative value of $n = 0.33$, calculate values of F_N for air ($k = 0.026$ W/m · K, $\beta = 0.0035$ K $^{-1}$, $\nu = 1.5 \times 10^{-5}$ m ^{2} /s, $Pr = 0.70$), water ($k = 0.600$ W/m · K, $\beta = 2.7 \times 10^{-4}$ K $^{-1}$, $\nu = 10^{-6}$ m ^{2} /s, $Pr = 5.0$), and a dielectric liquid ($k = 0.064$ W/m · K, $\beta = 0.0014$ K $^{-1}$, $\nu = 10^{-6}$ m ^{2} /s, $Pr = 25$). What fluid is the most effective cooling agent?
- 9.6** In many cases, we are concerned with free convection involving gases that are contained within sealed enclosures. Consider air at 27°C and pressures of 1, 10, and 100 bars. Determine the *figure of merit* described in Problem 9.5 for each of these three pressures. Which air pressure will provide the most effective cooling? *Hint:* See Problem 6.19.

Vertical Plates

- 9.7** Consider a large vertical plate with a uniform surface temperature of 100°C suspended in quiescent air at 25°C and atmospheric pressure.
- (a) Estimate the boundary layer thickness at a location 0.28 m measured from the lower edge.
- (b) What is the maximum velocity in the boundary layer at this location and at what position in the boundary layer does the maximum occur?
- (c) Using the similarity solution result, Equation 9.19, determine the heat transfer coefficient 0.25 m from the lower edge.
- (d) At what location on the plate measured from the lower edge will the boundary layer become turbulent?
- 9.8** For laminar free convection flow on a vertical plate, the recommended values of C and n for use in the correlation of Equation 9.24 are 0.59 and 1/4, respectively. Derive the values of C from the similarity solution, Equation 9.21, for $Pr = 0.01, 1, 10$, and 100.
- 9.9** Consider an array of vertical rectangular fins, which is to be used to cool an electronic device mounted in quiescent, atmospheric air at $T_\infty = 27^\circ\text{C}$. Each fin has $L = 20$ mm and $H = 150$ mm and operates at an approximately uniform temperature of $T_s = 77^\circ\text{C}$.



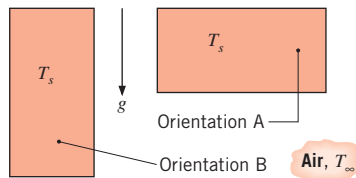
- (a) Viewing each fin surface as a vertical plate in an infinite, quiescent medium, briefly describe why there exists an optimum fin spacing S . Using Figure 9.4, estimate the optimum value of S for the prescribed conditions.
- (b) For the optimum value of S and a fin thickness of $t = 1.5$ mm, estimate the rate of heat transfer from the fins for an array of width $W = 355$ mm.
- 9.10** A number of thin plates are to be cooled by vertically suspending them in a water bath at a temperature of 20°C . If the plates are initially at 60°C and are 0.10 m long, what minimum spacing would prevent interference between their free convection boundary layers?
- 9.11** A square aluminum plate 5 mm thick and 150 mm on a side is heated while vertically suspended in quiescent air at 75°C . Determine the average heat transfer

coefficient for the plate when its temperature is 15°C by two methods: using results from the similarity solution to the boundary layer equations, and using results from an empirical correlation.

- 9.12** An aluminum alloy (2024) plate, heated to a uniform temperature of 227°C , is allowed to cool while vertically suspended in a room where the ambient air and surroundings are at 27°C . The plate is 0.3 m square with a thickness of 15 mm and an emissivity of 0.25 .

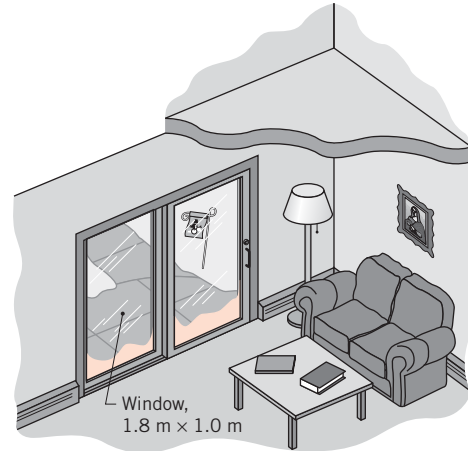
- Develop an expression for the time rate of change of the plate temperature, assuming the temperature to be uniform at any time.
- Determine the initial rate of cooling (K/s) when the plate temperature is 227°C .
- Justify the uniform plate temperature assumption.
- Compute and plot the temperature history of the plate from $t = 0$ to the time required to reach a temperature of 30°C . Compute and plot the corresponding variations in the convection and radiation heat transfer rates.

- 9.13** Consider a vertical plate of dimension $0.25\text{ m} \times 0.50\text{ m}$ that is at $T_s = 100^{\circ}\text{C}$ in a quiescent environment at $T_{\infty} = 20^{\circ}\text{C}$. In the interest of minimizing heat transfer from the plate, which orientation, (A) or (B), is preferred? Estimate the rate of heat transfer by free convection from the front surface of the plate when it is in the preferred orientation.



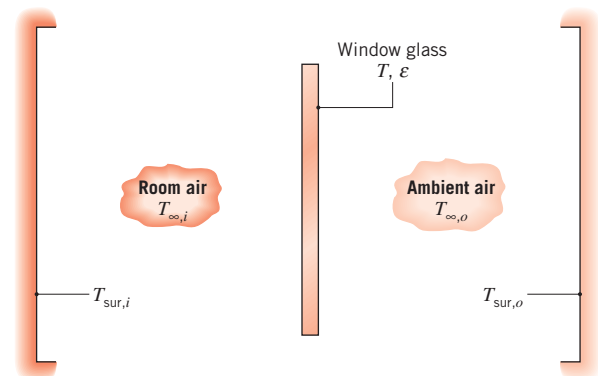
- 9.14** Show that the laminar, free convection correlations for a vertical, isothermal flat plate of height L can be applied to a vertical, isothermal cylinder of diameter D and height L if the boundary layer thickness at the top of the cylinder is approximately one-fourth the cylinder diameter. Determine the critical cylinder diameter, D_{crit} , below which the flat plate correlations cannot be applied when the surface is at $T_s = 35^{\circ}\text{C}$, the surrounding, quiescent air is at $T_{\infty} = 25^{\circ}\text{C}$, and $L = 0.75\text{ m}$. Calculate the rate of free convection heat transfer from the vertical cylinder wall corresponding to the critical cylinder diameter.

- 9.15** During a winter day, the window of a patio door with a height of 1.8 m and width of 1.0 m shows a frost line near its base. The room wall and air temperatures are 15°C .



- Explain why the window would show a frost layer at the base rather than at the top.
- Estimate the rate of heat loss through the window due to free convection and radiation. Assume the window has a uniform temperature of 0°C and the emissivity of the glass surface is 0.94 . If the room has electric baseboard heating, estimate the corresponding daily cost of the window heat loss for a utility rate of $0.18\text{ \$/kW} \cdot \text{h}$.

- 9.16** A vertical, thin pane of window glass that is 1 m on a side separates quiescent room air at $T_{\infty,i} = 20^{\circ}\text{C}$ from quiescent ambient air at $T_{\infty,o} = -20^{\circ}\text{C}$. The walls of the room and the external surroundings (landscape, buildings, etc.) are also at $T_{\text{sur},i} = 20^{\circ}\text{C}$ and $T_{\text{sur},o} = -20^{\circ}\text{C}$, respectively.



If the glass has an emissivity of $\epsilon = 1$, what is its temperature T ? What is the rate of heat loss through the glass?

- 9.17** Consider the conditions of Problem 9.16, but now allow for a difference between the inner and outer surface temperatures, $T_{s,i}$ and $T_{s,o}$, of the window. For a glass thickness and thermal conductivity of $t_g = 10\text{ mm}$ and $k_g = 1.4\text{ W/m} \cdot \text{K}$, respectively, evaluate $T_{s,i}$ and $T_{s,o}$. What is the rate of heat loss through the window?

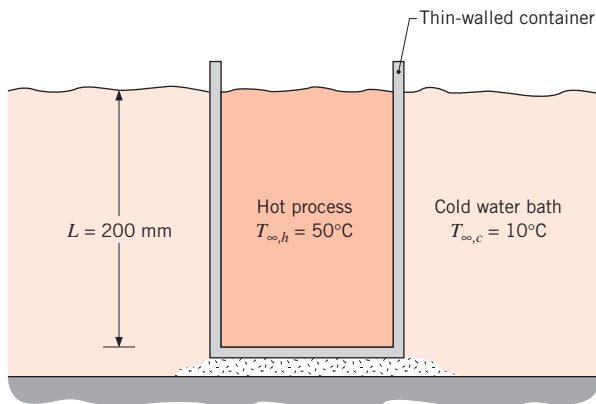
9.18 Consider laminar flow about a vertical isothermal plate of length L , providing an average heat transfer coefficient of $\bar{h}_L$. If the plate is divided into N smaller plates, each of length $L_N = L/N$, determine an expression for the ratio of the heat transfer coefficient averaged over the N plates to the heat transfer coefficient averaged over the single plate, $\bar{h}_{L,N}/\bar{h}_{L,1}$.

9.19 Consider the conveyor system described in Problem 7.19, but under conditions for which the conveyor is not moving and the air is quiescent. Radiation effects and interactions between boundary layers on adjoining surfaces may be neglected.

(a) For the prescribed plate dimensions and initial temperature, as well as the prescribed air temperature, what is the initial rate of heat transfer from one of the plates?

(b) How long does it take for a plate to cool from 300°C to 100°C? Comment on the assumption of negligible radiation.

9.20 A thin-walled container with a hot process fluid at 50°C is placed in a quiescent, cold water bath at 10°C. Heat transfer at the inner and outer surfaces of the container may be approximated by free convection from a vertical plate.



(a) Determine the overall heat transfer coefficient between the hot process fluid and the cold water bath. Assume the properties of the hot process fluid are those of water.

(b) Generate a plot of the overall heat transfer coefficient as a function of the hot process fluid temperature $T_{\infty,h}$ for the range 20 to 60°C, with all other conditions remaining the same.

9.21 Consider an experiment to investigate the transition to turbulent flow in a free convection boundary layer that develops along a vertical plate suspended in a large room. The plate is constructed of a thin heater that is sandwiched between two aluminum plates and may be assumed to be isothermal. The heated plate is 1 m high and 2 m wide. The quiescent air and the surroundings are both at 25°C.

(a) The exposed surfaces of the aluminum plate are painted with a very thin coating of high emissivity ($\epsilon = 0.95$) paint. Determine the electrical power that must be supplied to the heater to sustain the plate at a temperature of $T_s = 35^\circ\text{C}$. How much of the plate is exposed to turbulent conditions in the free convection boundary layer?

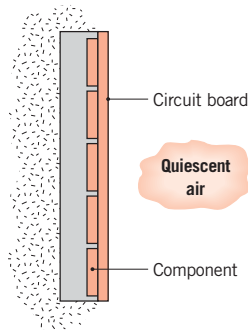
(b) The experimentalist speculates that the roughness of the paint is affecting the transition to turbulence in the boundary layer and decides to remove the paint and polish the aluminum surface ($\epsilon = 0.05$). If the same power is supplied to the plate as in part (a), what is the steady-state plate temperature? How much of the plate is exposed to turbulent conditions in the free convection boundary layer?

9.22 The vertical rear window of an automobile is of thickness $L = 8 \text{ mm}$ and height $H = 0.5 \text{ m}$ and contains fine-meshed heating wires that can induce nearly uniform volumetric heating, $\dot{q} \text{ (W/m}^3\text{)}$.

(a) Consider steady-state conditions for which the interior surface of the window is exposed to quiescent air at 10°C, while the exterior surface is exposed to air at -10°C moving in parallel flow over the surface with a velocity of 20 m/s. Determine the volumetric heating rate needed to maintain the interior window surface at $T_{s,i} = 15^\circ\text{C}$.

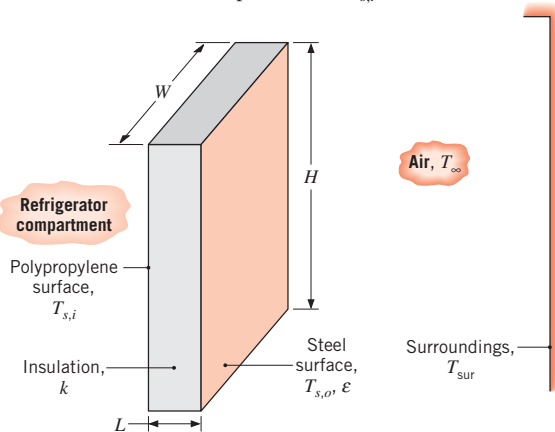
(b) The interior and exterior window temperatures, $T_{s,i}$ and $T_{s,o}$, depend on the compartment and ambient temperatures, $T_{\infty,i}$ and $T_{\infty,o}$, as well as on the velocity u_∞ of air flowing over the exterior surface and the volumetric heating rate $\dot{q}$. Subject to the constraint that $T_{s,i}$ is to be maintained at 15°C , we wish to develop guidelines for varying the heating rate in response to changes in $T_{\infty,i}$, $T_{\infty,o}$, and/or u_∞ . If $T_{\infty,i}$ is maintained at 10°C , how will $\dot{q}$ and $T_{s,o}$ vary with $T_{\infty,o}$ for $-25 \leq T_{\infty,o} \leq 5^\circ\text{C}$ and $u_\infty = 10, 20$, and 30 m/s ? If a constant vehicle speed is maintained, such that $u_\infty = 30 \text{ m/s}$, how will $\dot{q}$ and $T_{s,o}$ vary with $T_{\infty,i}$ for $5 \leq T_{\infty,i} \leq 20^\circ\text{C}$ and $T_{\infty,o} = -25, -10$, and 5°C ?

9.23 The components of a vertical circuit board, 150 mm on a side, dissipate 5 W. The back surface is well insulated and the front surface is exposed to quiescent air at 27°C.

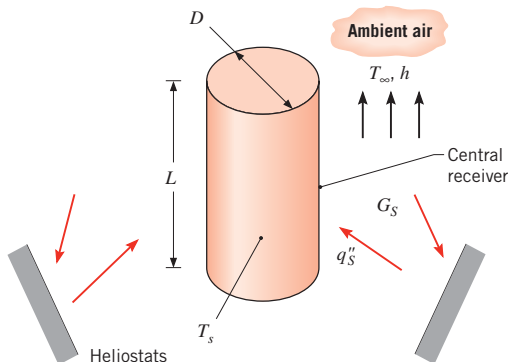


Assuming a uniform surface heat flux, what is the maximum temperature of the board? What is the temperature of the board for an isothermal surface condition?

- 9.24** A refrigerator door has a height and width of $H = 1$ m and $W = 0.65$ m, respectively, and is situated in a large room for which the air and walls are at $T_\infty = T_{\text{sur}} = 25^\circ\text{C}$. The door consists of a layer of polystyrene insulation ($k = 0.03$ W/m · K) sandwiched between thin sheets of steel ($\varepsilon = 0.6$) and polypropylene. Under normal operating conditions, the inner surface of the door is maintained at a fixed temperature of $T_{s,i} = 5^\circ\text{C}$.



- (a) Estimate the rate of heat gain through the door for the worst case condition corresponding to no insulation ($L = 0$).
- (b) Compute and plot the rate of heat gain and the outer surface temperature $T_{s,o}$ as a function of insulation thickness for $0 \leq L \leq 25$ mm.
- 9.25** In the *central receiver* concept of a solar power plant, many heliostats at ground level are used to direct a concentrated solar flux q_s'' to the receiver, which is positioned at the top of a tower. However, even with absorption of all the solar flux by the outer surface of the receiver, losses due to free convection and radiation reduce the collection efficiency below the maximum possible value of 100%. Consider a cylindrical receiver of diameter $D = 7$ m, length $L = 12$ m, and emissivity $\varepsilon = 0.20$.



- (a) If all of the solar flux is absorbed by the receiver and a surface temperature of $T_s = 800$ K is maintained, what is the rate of heat loss from the receiver? The ambient air is quiescent at a temperature of $T_\infty = 300$ K, and irradiation from the surroundings may be neglected. If the corresponding value of the solar flux is $q_s'' = 10^5$ W/m², what is the collector efficiency?
- (b) The surface temperature of the receiver is affected by design and operating conditions within the power plant. Over the range from 600 to 1000 K, plot the variation of the convection, radiation, and total heat rates as a function of T_s . For a fixed value of $q_s'' = 10^5$ W/m², plot the corresponding variation of the receiver efficiency.

Horizontal and Inclined Plates

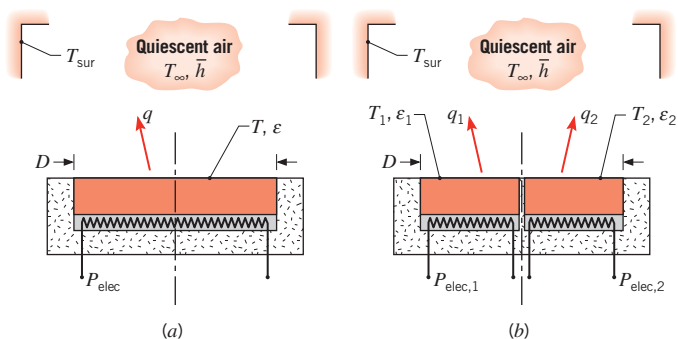
- 9.26** Airflow through a long, 0.3-m-square air conditioning duct maintains the outer duct surface temperature at 15°C . If the horizontal duct is uninsulated and exposed to air at 35°C in the crawlspace beneath a home, what is the heat gain per unit length of the duct? Neglect radiation.
- 9.27** Consider the conditions of Example 9.3, including the effect of adding insulation of thickness t and thermal conductivity $k = 0.035$ W/m · K to the duct. We wish to now include the effect of radiation on the outer surface temperatures and the total rate of heat loss per unit length of duct.
- (a) If $T_{s,1} = 45^\circ\text{C}$, $t = 25$ mm, $\varepsilon = 1$, and $T_{\text{sur}} = 288$ K, what are the temperatures of the side, top, and bottom surfaces? What are the corresponding rates of heat loss per unit length of duct?
- (b) For the top surface, compute and plot $T_{s,2}$ and q' as a function of insulation thickness for $0 \leq t \leq 50$ mm. The exposed duct surface ($t = 0$) may also be assumed to have an emissivity of $\varepsilon = 1$.

- 9.28** An electrical heater in the form of a horizontal disk of 500-mm diameter is used to heat the bottom of a tank filled with engine oil at a temperature of 10°C . Calculate the power required to maintain the heater surface temperature at 65°C .

- 9.29** Consider a horizontal 5-mm-thick, 100-mm-long straight fin fabricated from plain carbon steel ($k = 57$ W/m · K, $\varepsilon = 0.5$). The base of the fin is maintained at 100°C , while the quiescent ambient air and the surroundings are at 25°C . Assume the fin tip is adiabatic, and estimate the fin heat rate per unit width, q_f' . Use an average fin surface temperature of 80°C to estimate the free convection coefficient and the linearized radiation coefficient. How sensitive is your estimate to the choice of the average fin surface temperature?

9.30 A hot, 0.5-m-diameter, 35-mm-thick aluminum alloy disk is quenched from an initial temperature of $T_i = 400^\circ\text{C}$ in a large oil bath of temperature $T_\infty = 35^\circ\text{C}$. As it is quenched, the disk rests horizontally on the bottom of the well-insulated bath container. Use the results of Section 5.3.3 to determine the time needed to reduce the disk temperature to 100°C . The metal properties are $\rho = 1000 \text{ kg/m}^3$, $k = 185 \text{ W/m} \cdot \text{K}$, and $c = 775 \text{ J/kg} \cdot \text{K}$.

9.31 Convection heat transfer coefficients for a hot horizontal surface facing upward may be determined by a gage whose specific features depend on whether the temperature of the surroundings is known. For configuration A, a copper disk, which is electrically heated from below, is encased in an insulating material such that all of the heat is transferred by convection and radiation from the top surface. If the surface emissivity and the temperatures of the air and surroundings are known, the convection coefficient may be determined from measurement of the electrical power and the surface temperature of the disk. Configuration B is used in situations for which the temperature of the surroundings is not known. A thin, insulating strip separates semicircular disks with independent electrical heaters and different emissivities. If the emissivities and temperature of the air are known, the convection coefficient may be determined from measurement of the electrical power supplied to each of the disks in order to maintain them at a common temperature.



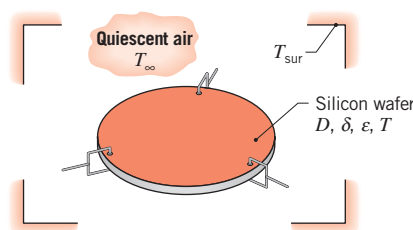
- (a) In an application of configuration A to a disk of diameter $D = 160 \text{ mm}$ and emissivity $\varepsilon = 0.8$, values of $P_{\text{elec}} = 10.8 \text{ W}$ and $T = 67^\circ\text{C}$ are measured for $T_\infty = T_{\text{sur}} = 27^\circ\text{C}$. What is the corresponding value of the average convection coefficient? How does it compare with predictions based on a standard correlation?
- (b) Now consider an application of configuration B for which $T_\infty = 17^\circ\text{C}$ and T_{sur} is unknown. With $D = 160 \text{ mm}$, $\varepsilon_1 = 0.8$, and $\varepsilon_2 = 0.1$, values of $P_{\text{elec},1} = 9.70 \text{ W}$ and $P_{\text{elec},2} = 5.67 \text{ W}$ are measured

when $T_1 = T_2 = 77^\circ\text{C}$. Determine the corresponding values of the convection coefficient and the temperature of the surroundings. How does the convection coefficient compare with predictions by an appropriate correlation?

9.32 Many laptop computers are equipped with thermal management systems that involve liquid cooling of the central processing unit (CPU), transfer of the heated liquid to the back of the laptop screen assembly, and dissipation of heat from the back of the screen assembly by way of a flat, isothermal heat spreader. The cooled liquid is recirculated to the CPU and the process continues. Consider an aluminum heat spreader that is of width $w = 275 \text{ mm}$ and height $L = 175 \text{ mm}$. The screen assembly is oriented at an angle $\theta = 30^\circ$ from the vertical direction, and the heat spreader is attached to the $t = 3\text{-mm}$ -thick plastic housing with a thermally conducting adhesive. The plastic housing has a thermal conductivity of $k = 0.21 \text{ W/m} \cdot \text{K}$ and emissivity of $\varepsilon = 0.85$. The contact resistance associated with the heat spreader-housing interface is $R''_{t,c} = 2.0 \times 10^{-4} \text{ m}^2 \cdot \text{K/W}$. If the CPU generates, on average, 15 W of thermal energy, what is the temperature of the heat spreader when $T_\infty = T_{\text{sur}} = 23^\circ\text{C}$? Which thermal resistance (contact, conduction, radiation, or free convection) is the largest?

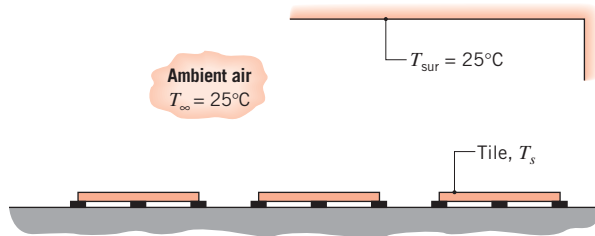
9.33 Consider the roof of the refrigerated truck compartment described in Problem 7.16, but under conditions for which the truck is parked ($V = 0$). All other conditions remain unchanged. For $\alpha_s = \varepsilon = 0.6$, determine the outer surface temperature, $T_{s,o}$, and the heat load imposed on the refrigeration system. *Hint:* Assume $T_{s,o} > T_\infty$ and $Ra_L > 10^7$.

9.34 At the end of its manufacturing process, a silicon wafer of diameter $D = 150 \text{ mm}$, thickness $\delta = 1 \text{ mm}$, and emissivity $\varepsilon = 0.65$ is at an initial temperature of $T_i = 325^\circ\text{C}$ and is allowed to cool in quiescent, ambient air and large surroundings for which $T_\infty = T_{\text{sur}} = 25^\circ\text{C}$.

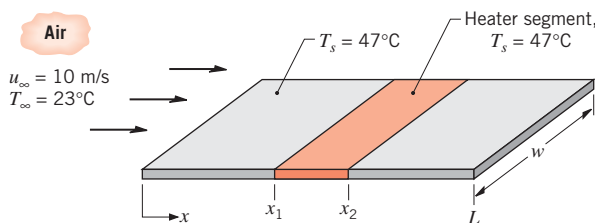


- (a) What is the initial rate of cooling?
- (b) How long does it take for the wafer to reach a temperature of 50°C ? Comment on how the relative effects of convection and radiation vary with time during the cooling process.

- 9.35** A 200-mm-square, 10-mm-thick tile has the thermo-physical properties of Pyrex ($\epsilon = 0.80$) and emerges from a curing process at an initial temperature of $T_i = 140^\circ\text{C}$. The backside of the tile is insulated while the upper surface is exposed to ambient air and surroundings at 25°C .

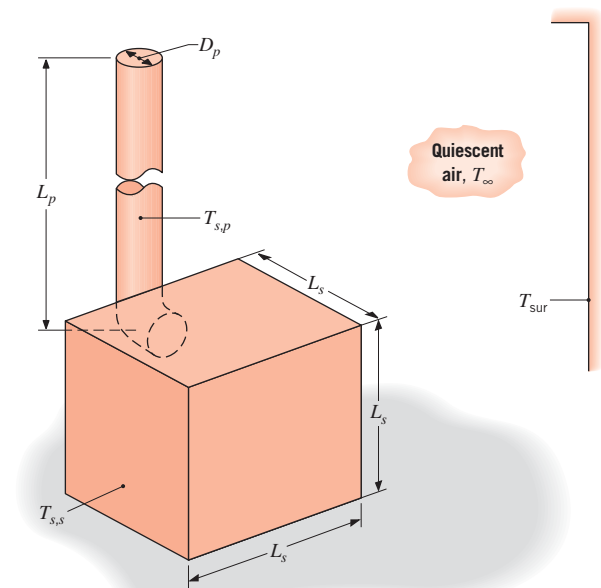


- (a) Estimate the time required for the tile to cool to a final, safe-to-touch temperature of $T_f = 40^\circ\text{C}$. Use an average tile surface temperature of $\bar{T} = (T_i + T_f)/2$ to estimate the average free convection coefficient and the linearized radiation coefficient. How sensitive is your estimate to the assumed value for $\bar{T}$?
- (b) Estimate the required cooling time if ambient air is blown in parallel flow over the tile with a velocity of 10 m/s.
- 9.36** A highly polished aluminum plate of length 0.5 m and width 0.2 m is subjected to an airstream at a temperature of 23°C and a velocity of 10 m/s. Because of upstream conditions, the flow is turbulent over the entire length of the plate. A series of segmented, independently controlled heaters is attached to the lower side of the plate to maintain approximately isothermal conditions over the entire plate. The electrical heater covering the section between the positions $x_1 = 0.2$ m and $x_2 = 0.3$ m is shown in the schematic.



- (a) Estimate the electrical power that must be supplied to the designated heater segment to maintain the plate surface temperature at $T_s = 47^\circ\text{C}$.
- (b) If the blower that maintains the airstream velocity over the plate malfunctions, but the power to the heaters remains constant, estimate the surface temperature of the designated segment. Assume that the ambient air is extensive, quiescent, and at 23°C .

- 9.37** Certain wood stove designs rely exclusively on heat transfer by radiation and natural convection to the surroundings. Consider a stove that forms a cubical enclosure, $L_s = 1$ m on a side, in a large room. The exterior walls of the stove have an emissivity of $\epsilon = 0.8$ and are at an operating temperature of $T_{s,s} = 500$ K.



- The stove pipe, which may be assumed to be isothermal at an operating temperature of $T_{s,p} = 400$ K, has a diameter of $D_p = 0.25$ m and a height of $L_p = 2$ m, extending from stove to ceiling. The stove is in a large room whose air and walls are at $T_\infty = T_{\text{sur}} = 300$ K. Neglecting heat transfer from the small horizontal section of the pipe and radiation exchange between the pipe and stove, estimate the rate at which heat is transferred from the stove and pipe to the surroundings.

- 9.38** A plate 1 m $\times$ 1 m, inclined at an angle of 45° , is exposed to a net radiation heat flux of 300 W/m^2 at its bottom surface. If the top surface of the plate is well insulated, estimate the temperature the plate reaches when the ambient air is quiescent and at a temperature of 0°C .

Horizontal Cylinders and Spheres

- 9.39** A horizontal uninsulated steam pipe passes through a large room whose walls and ambient air are at 300 K. The pipe of 125-mm diameter has an emissivity of 0.85 and an outer surface temperature of 373 K. Calculate the rate of heat loss per unit length from the pipe.

9.40 As discussed in Section 5.2, the lumped capacitance approximation may be applied if $B_i < 0.1$, and, when implemented in a conservative fashion for a long cylinder, the characteristic length is the cylinder radius. After its extrusion, a long glass rod of diameter $D = 15$ mm is suspended horizontally in a room and cooled from its initial temperature by natural convection and radiation. At what rod temperatures may the lumped capacitance approximation be applied? The temperature of the quiescent air is the same as that of the surroundings, $T_\infty = T_{\text{sur}} = 27^\circ\text{C}$, and the glass emissivity is $\varepsilon = 0.94$.

9.41 A long, uninsulated steam line with a diameter of 100 mm and a surface emissivity of 0.8 transports steam at 150°C and is exposed to atmospheric air and large surroundings at an equivalent temperature of 20°C .

- Calculate the rate of heat loss per unit length for a calm day.
- Calculate the rate of heat loss on a breezy day when the wind speed is 8 m/s.
- For the conditions of part (a), calculate the rate of heat loss with a 20-mm-thick layer of insulation ($k = 0.08$ W/m · K). Would the heat loss change significantly with an appreciable wind speed?

9.42 Consider Problem 8.38. A more realistic solution would account for the resistance to heat transfer due to free convection in the paraffin during melting. Assuming the tube surface to have a uniform temperature of 55°C and the paraffin to be an infinite, quiescent liquid, determine the convection coefficient associated with the outer surface. Using this result and recognizing that the tube surface temperature is not known, determine the water outlet temperature, the total heat transfer rate, and the time required to completely liquefy the paraffin, for the prescribed conditions. Thermophysical properties associated with the liquid state of the paraffin are $k = 0.15$ W/m · K, $\beta = 8 \times 10^{-4}$ K $^{-1}$, $\rho = 770$ kg/m ^{3} , $\nu = 5 \times 10^{-6}$ m ^{2} /s, and $\alpha = 8.85 \times 10^{-8}$ m ^{2} /s.

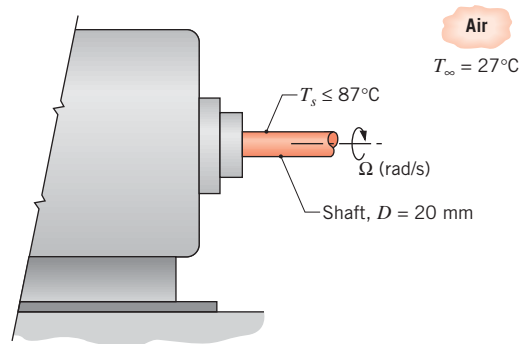
9.43 Saturated steam at 4 bars absolute pressure with a mean velocity of 3 m/s flows through a horizontal pipe whose inner and outer diameters are 55 and 65 mm, respectively. The heat transfer coefficient for the steam flow is known to be 11,000 W/m ^{2} · K.

- If the pipe is covered with a 25-mm-thick layer of 85% magnesia insulation and is exposed to atmospheric air at 25°C , determine the rate of heat transfer by free convection to the room per unit length of the pipe. If the steam is saturated at the inlet of the pipe, estimate its quality at the outlet of a pipe 30 m long.
- Net radiation to the surroundings also contributes to heat loss from the pipe. If the insulation has a

surface emissivity of $\varepsilon = 0.8$ and the surroundings are at $T_{\text{sur}} = T_\infty = 25^\circ\text{C}$, what is the rate of heat transfer to the room per unit length of pipe? What is the quality of the outlet flow?

- (c) The heat loss may be reduced by increasing the insulation thickness and/or reducing its emissivity. What is the effect of increasing the insulation thickness to 50 mm if $\varepsilon = 0.8$? Of decreasing the emissivity to 0.2 if the insulation thickness is 25 mm? Of reducing the emissivity to 0.2 and increasing the insulation thickness to 50 mm?

9.44 The maximum surface temperature of the 20-mm-diameter shaft of a motor operating in ambient air at 27°C should not exceed 87°C . Because of power dissipation within the motor housing, it is desirable to reject as much heat as possible through the shaft to the ambient air. In this problem, we will investigate several methods for heat removal.



- (a) For rotating cylinders, a suitable correlation for estimating the convection coefficient is of the form

$$\overline{Nu}_D = 0.133 Re_D^{2/3} Pr^{1/3}$$

$$(Re_D < 4.3 \times 10^5, \quad 0.7 < Pr < 670)$$

where $Re_D \equiv \Omega D^2/\nu$ and Ω is the rotational velocity (rad/s). Determine the convection coefficient and the maximum heat rate per unit length as a function of rotational speed in the range from 5000 to 15,000 rpm.

- Estimate the free convection coefficient and the maximum heat rate per unit length for the stationary shaft. Mixed free and forced convection effects may become significant for $Re_D < 4.7(Gr_D^3/Pr)^{0.137}$. Are free convection effects important for the range of rotational speeds designated in part (a)?
- Assuming the emissivity of the shaft is 0.8 and the surroundings are at the ambient air temperature, is radiation exchange important?
- If ambient air is in cross flow over the shaft, what air velocities are required to remove the heat rates determined in part (a)?

9.45 Consider a horizontal pin fin of 6-mm diameter and 60-mm length fabricated from plain carbon steel ($k = 57 \text{ W/m} \cdot \text{K}$, $\varepsilon = 0.5$). The base of the fin is maintained at 150°C , while the quiescent ambient air and the surroundings are at 25°C . Assume the fin tip is adiabatic.

(a) Estimate the fin heat rate, q_f . Use an average fin surface temperature of 125°C in estimating the free convection coefficient and the linearized radiation coefficient. How sensitive is this estimate to your choice of the average fin surface temperature?

(b) Use the finite-difference method of solution to obtain q_f when the convection and radiation coefficients are based on local, rather than average, temperatures for the fin. How does your result compare with the analytical solution of part (a)?

9.46 The concept of a critical insulation radius was introduced in Example 3.6. Consider the thin-walled copper tube and insulation of the example. Now, the tube temperature is -10°C and it is suspended horizontally in quiescent air at 25°C . Neglecting radiation, determine the heat transfer rate per unit tube length for an insulation thickness of 10 mm. Graph the conduction, convection, and total thermal resistances for a unit length for insulation thickness $r - r_i$ over the range $0 \leq r - r_i \leq 50 \text{ mm}$, and compare your results to that of the example. Repeat the calculation and the graphing exercise for a tube diameter of 1 mm. For each case, does a critical insulation radius exist? Why or why not? Evaluate air properties at 285 K .

9.47 Consider the hot water pipe of Problem 7.42, but under conditions for which the ambient air is not in cross flow over the pipe and is, instead, quiescent. Accounting for the effect of radiation with a pipe emissivity of $\varepsilon_p = 0.6$, what is the corresponding daily cost of heat loss per unit length of the uninsulated pipe?

9.48 Common practice in chemical processing plants is to clad pipe insulation with a durable, thick aluminum foil. The functions of the foil are to confine the batt insulation and to reduce heat transfer by radiation to the surroundings. Because of the presence of chlorine (at chlorine or seaside plants), the aluminum foil surface, which is initially bright, becomes etched with in-service time. Typically, the emissivity might change from 0.12 at installation to 0.36 with extended service. For a 300-mm-diameter foil-covered pipe whose surface temperature is 90°C , will this increase in emissivity due to degradation of the foil finish have a significant effect on heat loss from the pipe? Consider two cases with surroundings and ambient air at 25°C : (a) quiescent air and (b) a cross-wind velocity of 10 m/s .

9.49 Consider the electrical heater of Problem 7.36. If the blower were to malfunction, terminating airflow while the heater continued to operate at 1000 W/m , what temperature would the heater assume? How long would it take to come within 10°C of this temperature? Allow for radiation exchange between the heater ($\varepsilon = 0.8$) and the duct walls, which are also at 27°C .

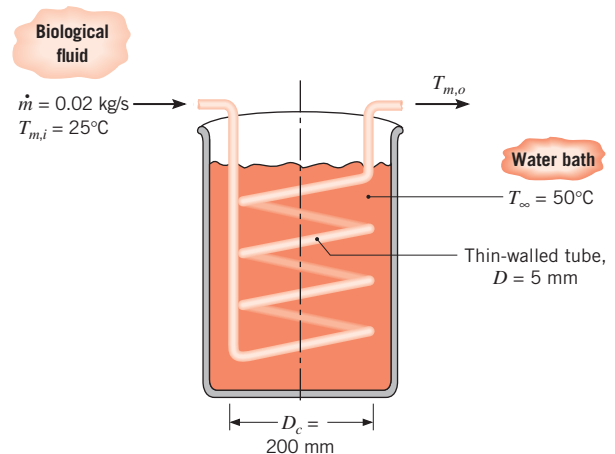
9.50 A billet of stainless steel, AISI 316, with a diameter of 150 mm and a length of 500 mm emerges from a heat treatment process at 200°C and is placed in an unstirred oil bath maintained at 20°C .

(a) Determine whether it is advisable to position the billet in the bath with its centerline horizontal or vertical in order to decrease the cooling time.

(b) Estimate the time for the billet to cool to 30°C for the preferred arrangement.

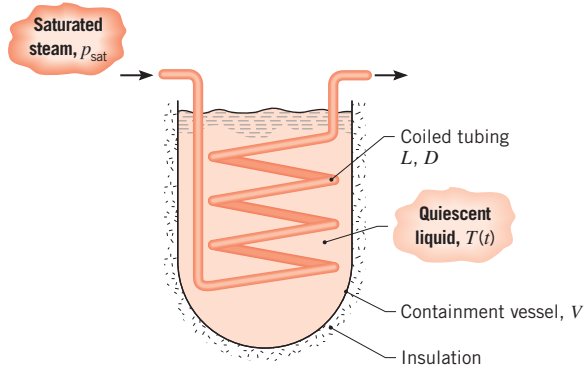
9.51 A biological fluid moves at a flow rate of $\dot{m} = 0.02 \text{ kg/s}$ through a coiled, thin-walled, 5-mm-diameter tube submerged in a large water bath maintained at 50°C . The fluid enters the tube at 25°C .

(a) Estimate the length of the tube and the number of coil turns required to provide an exit temperature of $T_{m,o} = 38^\circ\text{C}$ for the biological fluid. Assume that the water bath is an extensive, quiescent medium, that the coiled tube approximates a horizontal tube, and that the biological fluid has the thermophysical properties of water.



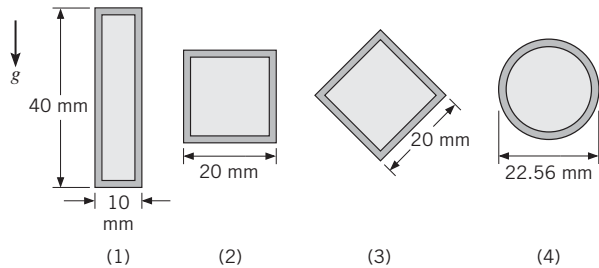
(b) The flow rate through the tube is controlled by a pump that experiences throughput variations of approximately $\pm 10\%$ at any one setting. This condition is of concern to the project engineer because the corresponding variation of the exit temperature of the biological fluid could influence the downstream process. What variation would you expect in $T_{m,o}$ for a $\pm 10\%$ change in $\dot{m}$?

- 9.52** Consider a batch process in which 200 L of a pharmaceutical are heated from 25°C to 70°C by saturated steam condensing at 2.455 bars as it flows through a coiled tube of 15-mm diameter and 15-m length. At any time during the process, the liquid may be approximated as an infinite, quiescent medium of uniform temperature and may be assumed to have constant properties of $\rho = 1100 \text{ kg/m}^3$, $c = 2000 \text{ J/kg} \cdot \text{K}$, $k = 0.25 \text{ W/m} \cdot \text{K}$, $\nu = 4.0 \times 10^{-6} \text{ m}^2/\text{s}$, $Pr = 10$, and $\beta = 0.002 \text{ K}^{-1}$. The thermal resistances of the condensing steam and tube wall may be neglected.



- (a) What is the initial rate of heat transfer to the pharmaceutical?
- (b) Neglecting heat transfer between the tank and its surroundings, how long does it take to heat the pharmaceutical to 70°C? Plot the corresponding variation with time of the fluid temperature and the convection coefficient at the outer surface of the tube. How much steam is condensed during the heating process?
- 9.53** In the analytical treatment of the fin with uniform cross-sectional area, it was assumed that the convection heat transfer coefficient is constant along the length of the fin. Consider an AISI 316 steel fin of 6-mm diameter and 50-mm length (with insulated tip) operating under conditions for which $T_b = 125^\circ\text{C}$, $T_\infty = 27^\circ\text{C}$, $T_{\text{sur}} = 27^\circ\text{C}$, and $\varepsilon = 0.6$.
- (a) Estimate average values of the fin heat transfer coefficients for free convection (h_c) and radiation exchange (h_r). Use these values to predict the tip temperature and fin effectiveness.
- (b) Use a numerical method of solution to estimate the foregoing parameters when the convection and radiation coefficients are based on local, rather than average, values for the fin.
- 9.54** A hot fluid at 35°C is to be transported through a tube horizontally positioned in quiescent air at 25°C. Which

of the tube shapes, each of equal cross-sectional area, would you use in order to minimize heat losses to the ambient air by free convection?



Use the following correlation of Lienhard [Int. J. Heat Mass Transfer, **16**, 2121, 1973] to approximate the laminar convection coefficient for an immersed body on which the boundary layer does not separate from the surface,

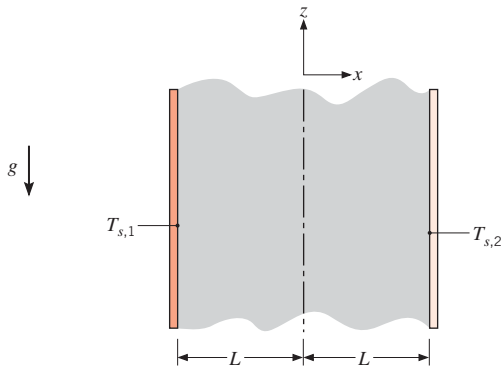
$$\overline{Nu}_l = 0.52 Ra_l^{1/4}$$

The characteristic length l is the length of travel of the fluid in the boundary layer across the shape surface.

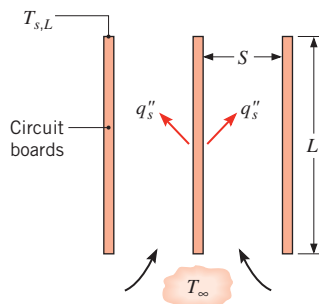
- 9.55** Consider a 2-mm-diameter sphere immersed in a fluid at 300 K and 1 atm.
- (a) If the fluid around the sphere is quiescent and extensive, show that the conduction limit of heat transfer from the sphere can be expressed as $Nu_{D,\text{cond}} = 2$. Hint: Begin with the expression for the thermal resistance of a hollow sphere, Equation 3.41, letting $r_2 \rightarrow \infty$, and then expressing the result in terms of the Nusselt number.
- (b) Considering free convection, at what surface temperature will the Nusselt number be twice that for the conduction limit? Consider air and water as the fluids.
- (c) Considering forced convection, at what velocity will the Nusselt number be twice that for the conduction limit? Consider air and water as the fluids.
- 9.56** A sphere of 30-mm diameter contains an embedded electrical heater. Calculate the power required to maintain the surface temperature at 89°C when the sphere is exposed to a quiescent medium at 25°C for: (a) air at atmospheric pressure, (b) water, and (c) engine oil.

Parallel Plate Channels

- 9.57** Consider two long vertical plates maintained at uniform temperatures $T_{s,1} > T_{s,2}$. The plates are open at their ends and are separated by the distance $2L$.



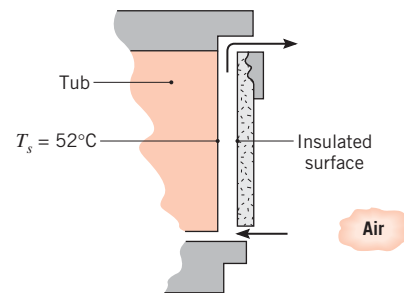
- (a) Sketch the velocity distribution in the space between the plates.
 - (b) Write appropriate forms of the continuity, momentum, and energy equations for laminar flow between the plates.
 - (c) Evaluate the temperature distribution, and express your result in terms of the mean temperature, $T_m = (T_{s,1} + T_{s,2})/2$.
 - (d) Estimate the vertical pressure gradient by assuming the density to be a constant ρ_m corresponding to T_m . Substituting from the Boussinesq approximation, obtain the resulting form of the momentum equation.
 - (e) Determine the velocity distribution.
- 9.58** Consider the conditions of Problem 9.9, but now view the problem as one involving free convection between vertical, parallel plate channels. What is the optimum fin spacing S ? For this spacing and the prescribed values of t and W , what is the rate of heat transfer from the fins?
- 9.59** A vertical array of circuit boards is immersed in quiescent ambient air at $T_\infty = 17^\circ\text{C}$. Although the components protrude from their substrates, it is reasonable, as a first approximation, to assume *flat* plates with uniform surface heat flux q_s'' . Consider boards of length and width $L = W = 0.4$ m and spacing $S = 25$ mm. If the maximum allowable board temperature is 77°C , what is the maximum allowable power dissipation per board?



9.60 Determined to reduce the \$7 per week cost associated with heat loss through their patio window by convection and radiation, the tenants of Problem 9.15 cover the inside of the window with a 50-mm-thick sheet of extruded insulation. Because they are not very handy around the house, the insulation is installed poorly, resulting in an $S = 5$ -mm gap between the extruded insulation and the window pane, allowing the room air to infiltrate into the space between the pane and the insulation.

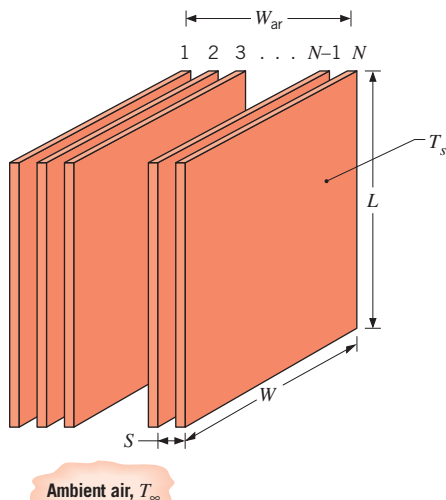
- (a) Determine the rate of heat loss through the window and associated weekly cost with the ill-fitting insulation in place. The insulation will significantly reduce the radiation losses through the window. Losses will be due almost entirely to convection.
- (b) Plot the rate of heat loss through the patio window as a function of the gap spacing for $1 \text{ mm} \leq S \leq 20 \text{ mm}$.

9.61 The front door of a dishwasher of width 580 mm has a vertical air vent that is 500 mm in height with a 20-mm spacing between the inner tub operating at 52°C and an outer plate that is thermally insulated.



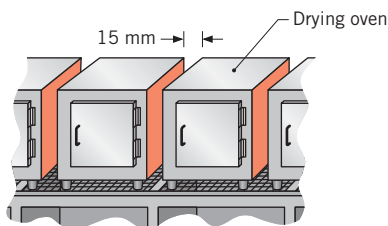
- (a) Determine the rate of heat loss from the tub surface when the ambient air is 27°C .
- (b) A change in the design of the door provides the opportunity to increase or decrease the 20-mm spacing by 10 mm. What recommendations would you offer with regard to how the change in spacing will alter heat losses?

9.62 A natural convection air heater consists of an array of parallel, equally spaced vertical plates, which may be maintained at a fixed temperature T_s by embedded electrical heaters. The plates are of length and width $L = W = 300$ mm and are in quiescent, atmospheric air at $T_\infty = 20^\circ\text{C}$. The total width of the array cannot exceed a value of $W_{\text{ar}} = 150$ mm.



For $T_s = 75^\circ\text{C}$, what is the plate spacing S that maximizes heat transfer from the array? For this spacing, how many plates comprise the array and what is the corresponding rate of heat transfer from the array?

- 9.63** A bank of drying ovens is mounted on a rack in a room with an ambient air temperature of 27°C . The cubical ovens are 500 mm to a side, and the spacing between the ovens is 15 mm.

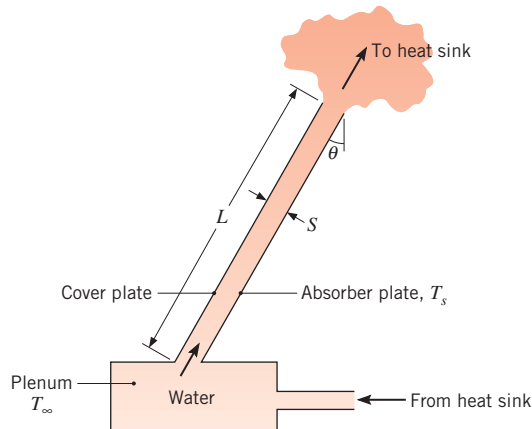


- (a) Estimate the rate of heat loss from a facing side of an oven when its surface temperature is 47°C .

- (b) Explore the effect of the spacing on the rate of heat loss. At what spacing is the rate of heat loss a maximum? Describe the boundary layer behavior for this condition. Can this condition be analyzed by treating the side of an oven as an isolated vertical plate?

- 9.64** A solar collector consists of a parallel plate channel that is connected to a water storage plenum at the bottom and to a heat sink at the top. The channel is inclined $\theta = 30^\circ$ from the vertical and has a transparent cover plate. Solar radiation transmitted through the cover plate and the water maintains the isothermal absorber plate at a temperature $T_s = 47^\circ\text{C}$, while water returned to the reservoir from the heat sink is at $T_\infty = 27^\circ\text{C}$. The system operates as a *thermosyphon*, for which water flow is

driven exclusively by buoyancy forces. The plate spacing and length are $S = 15\text{ mm}$ and $L = 1.2\text{ m}$.



Assuming the cover plate to be adiabatic with respect to convection heat transfer to or from the water, estimate the rate of heat transfer per unit width normal to the flow direction (W/m) from the absorber plate to the water.

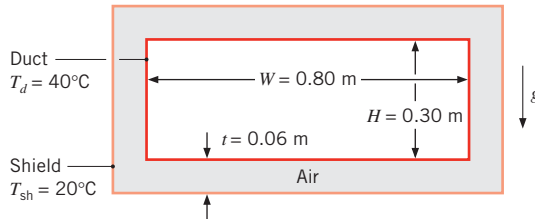
Rectangular Cavities

- 9.65** As is evident from the property data of Tables A.3 and A.4, the thermal conductivity of glass at room temperature is more than 50 times larger than that of air. It is therefore desirable to use windows of double-pane construction, for which the two panes of glass enclose an air space. If heat transfer across the air space is by conduction, the corresponding thermal resistance may be increased by increasing the thickness L of the space. However, there are limits to the efficacy of such a measure, since convection currents are induced if L exceeds a critical value, beyond which the thermal resistance decreases.

Consider atmospheric air enclosed by vertical panes at temperatures of $T_1 = 22^\circ\text{C}$ and $T_2 = -20^\circ\text{C}$. If the critical Rayleigh number for the onset of convection is $Ra_L \approx 2000$, what is the maximum allowable spacing for conduction across the air? How is this spacing affected by the temperatures of the panes? How is it affected by the pressure of the air, as, for example, by partial evacuation of the space?

- 9.66** A building window pane that is 1.2 m high and 0.8 m wide is separated from the ambient air by a storm window of the same height and width. The air space between the two windows is 0.06 m thick. If the building and storm windows are at 20 and -10°C , respectively, what is the rate of heat loss by free convection across the air space?

- 9.67** To reduce heat losses, a horizontal rectangular duct that is $W = 0.80$ m wide and $H = 0.3$ m high is encased in a metal radiation shield. The duct wall and shield are separated by an air gap of thickness $t = 0.06$ m. For a duct wall temperature of $T_d = 40^\circ\text{C}$ and a shield temperature of $T_{sh} = 20^\circ\text{C}$, determine the rate of convection heat loss per unit length from the duct.



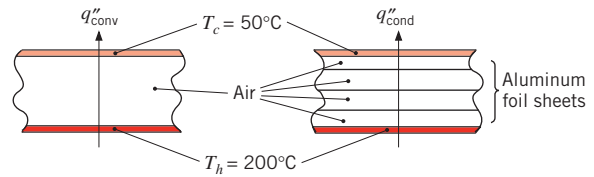
- 9.68** The absorber plate and the adjoining cover plate of a flat-plate solar collector are at 70 and 35°C , respectively, and are separated by an air space of 0.05 m. What is the rate of free convection heat transfer per unit surface area between the two plates if they are inclined at an angle of 60° from the horizontal?

- 9.69** Consider a thermal storage system in which the phase change material (paraffin) is housed in a large container whose bottom, horizontal surface is maintained at $T_s = 50^\circ\text{C}$ by warm water delivered from a solar collector.

- Neglecting the change in sensible energy of the liquid phase, estimate the amount of paraffin that is melted over a five-hour period beginning with an initial liquid layer at the bottom of the container of thickness $s_i = 10$ mm. The paraffin of Problems 8.38 and 9.42 is used as the phase change material and is initially at the phase change temperature, $T_{mp} = 27.4^\circ\text{C}$. The bottom surface area of the container is $A = 2.5$ m².
- Compare the amount of energy needed to melt the paraffin to the amount of energy required to increase the temperature of the same amount of liquid from the phase change temperature to the average liquid temperature, $(T_s + T_{mp})/2$.
- Neglecting the change in sensible energy of the liquid phase, estimate the amount of paraffin that would melt over a five-hour time period if the hot plate is placed at the top of the container and $s_i = 10$ mm.

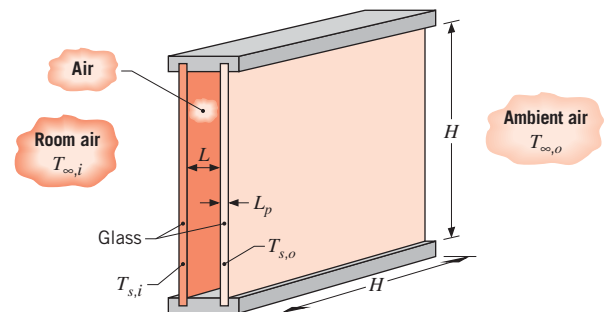
- 9.70** A 50-mm-thick air gap separates two horizontal metal plates that form the top surface of an industrial furnace. The bottom plate is at $T_h = 200^\circ\text{C}$ and the top plate is at $T_c = 50^\circ\text{C}$. The plant operator wishes to provide insulation between the plates to minimize heat loss. The relatively hot temperatures preclude use of foamed or felt insulation materials. Evacuated insulation materials cannot be used due to the harsh industrial environment

and their expense. A young engineer suggests that equally spaced, thin horizontal sheets of aluminum foil may be inserted in the gap to eliminate natural convection and minimize heat loss through the air gap.



- Determine the convective heat flux across the gap when no insulation is in place.
- Determine the minimum number of foil sheets that must be inserted in the gap to eliminate free convection.
- Determine the conduction heat flux across the air gap with the foil sheets in place.

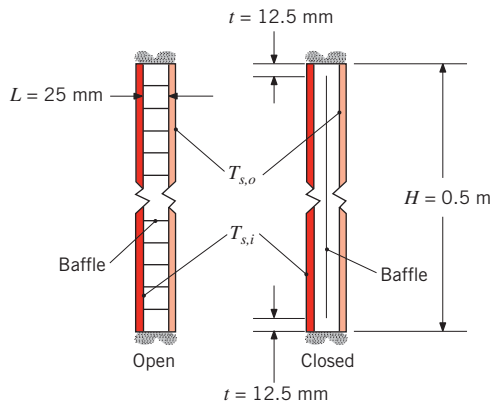
- 9.71** A vertical, double-pane window, which is 1 m on a side and has a 25-mm gap filled with atmospheric air, separates quiescent room air at $T_{\infty,i} = 20^\circ\text{C}$ from quiescent ambient air at $T_{\infty,o} = -20^\circ\text{C}$. Radiation exchange between the window panes, as well as between each pane and its surroundings, may be neglected.



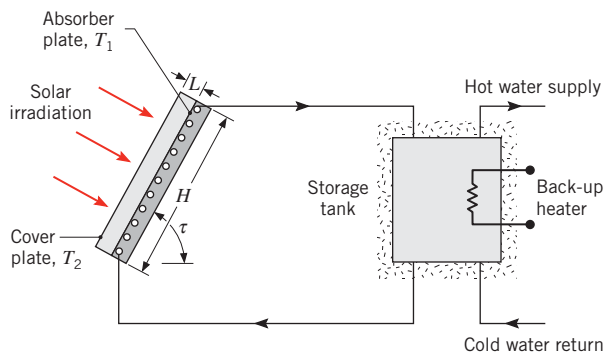
- Neglecting the thermal resistance associated with conduction heat transfer across each pane, determine the corresponding temperature of each pane and the rate of heat transfer through the window.
- Comment on the validity of neglecting the conduction resistance of the panes if each is of thickness $L_p = 6$ mm.

- 9.72** Consider window blinds that are installed in the air space between the two panes of a vertical double-pane window. The window is $H = 0.5$ m high and $w = 0.5$ m wide, and includes $N = 19$ individual blinds that are each $L = 25$ mm wide. When the blinds are open, 20 smaller, square enclosures are formed along the height of the window. In the closed position, the blinds form a nearly continuous sheet with two $t = 12.5$ mm open gaps at the top and bottom of the enclosure. Determine

the convection heat transfer rate between the inner pane, which is held at $T_{s,i} = 20^\circ\text{C}$, and the outer pane, which is at $T_{s,o} = -20^\circ\text{C}$, when the blinds are in the open and closed positions, respectively. Explain why the closed blinds have little effect on the convection heat transfer rate across the cavity.



- 9.73** A solar water heater consists of a flat-plate collector that is coupled to a storage tank. The collector consists of a transparent cover plate and an absorber plate that are separated by an air gap.



Although much of the solar energy collected by the absorber plate is transferred to a working fluid passing through a coiled tube brazed to the back of the absorber, some of the energy is lost by free convection and net radiation transfer across the air gap. In Chapter 13, we will evaluate the contribution of radiation exchange to this loss. For now, we restrict our attention to the free convection effect.

- (a) Consider a collector that is inclined at an angle of $\tau = 60^\circ$ and has dimensions of $H = w = 2$ m on a side, with an air gap of $L = 30$ mm. If the absorber and cover plates are at $T_1 = 70^\circ\text{C}$ and $T_2 = 30^\circ\text{C}$, respectively, what is the rate of heat transfer by free convection from the absorber plate?

- (b) The heat loss by free convection depends on the spacing between the plates. Compute and plot the

rate of heat loss as a function of spacing for $5 \leq L \leq 50$ mm. Is there an optimum spacing?

Concentric Cylinders and Spheres

- 9.74** Consider the cylindrical, 0.12-m-diameter radiation shield of Example 9.5 that is installed concentric with a 0.10-m-diameter tube carrying steam. The spacing provides an air gap of $L = 10$ mm.

- (a) Calculate the rate of heat loss per unit length of the tube by convection when a second shield of 0.14-m diameter is installed, with the second shield maintained at 35°C . Compare the result to that for the single shield of the example.
- (b) In the two-shield configuration of part (a), the air gaps formed by the annular concentric tubes are $L = 10$ mm. Calculate the rate of heat loss per unit length if the gap dimension is $L = 15$ mm. Do you expect the heat loss to increase or decrease?

- 9.75** The effective thermal conductivity k_{eff} for concentric cylinders and concentric spheres is provided in Equations 9.59 and 9.62, respectively. Derive expressions for the critical Rayleigh numbers associated with the cylindrical and spherical geometries, $Ra_{c,\text{crit}}$ and $Ra_{s,\text{crit}}$, respectively, below which k_{eff} is minimized. Evaluate $Ra_{c,\text{crit}}$ and $Ra_{s,\text{crit}}$ for air, water, and glycerin at a mean temperature of 300 K. For specified inner and outer surface temperatures and inner cylinder or sphere radii, comment on the heat transfer rate for outer cylinder or sphere radii corresponding to $Ra_{c,\text{crit}}$ and $Ra_{s,\text{crit}}$, respectively.

- 9.76** A solar collector design consists of an inner tube enclosed concentrically in an outer tube that is transparent to solar radiation. The tubes are thin walled with inner and outer diameters of 0.08 and 0.10 m, respectively. The annular space between the tubes is completely enclosed and filled with air at atmospheric pressure. Under operating conditions for which the inner and outer tube surface temperatures are 80 and 30°C , respectively, what is the rate of convective heat loss per meter of tube length across the air space?

- 9.77** Graph the heat loss per unit length from the solar collector of Problem 9.76 over the range $0.1 \leq D_o \leq 0.25$ m assuming (i) conduction heat transfer across the annular space and (ii) convection heat transfer across the annular space. Determine the outer diameter that yields the minimum heat loss across the air space per unit collector length, and the value of this loss.

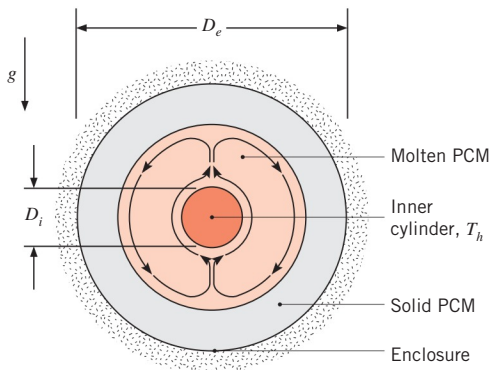
- 9.78** It has been proposed to use large banks of rechargeable, lithium ion batteries to power electric vehicles. The cylindrical batteries, each of which is of radius $r_i = 9$ mm

and length $L = 65$ mm, undergo exothermic electrochemical reactions while being discharged. Since excessively high temperatures damage the batteries, it is proposed to encase them in a phase change material that melts when the batteries discharge (and resolidifies when the batteries are charged; charging is associated with an endothermic electrochemical reaction). Consider the paraffin of Problems 8.38 and 9.42.

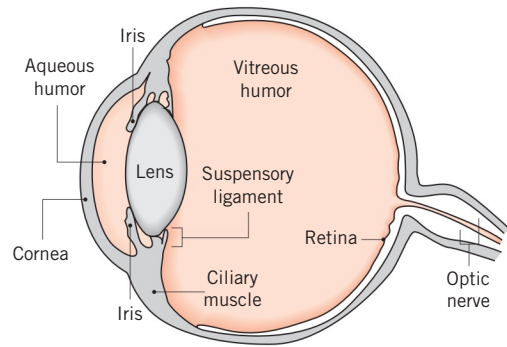
- At an instant in time during the discharge of a battery, liquid paraffin occupies an annular region of outer radius $r_o = 19$ mm around the battery, which is generating $\dot{E}_g = 1$ W of thermal energy. Determine the surface temperature of the battery.
- At the time of interest in part (a), what is the rate at which the liquid annulus radius is increasing?
- Plot the battery surface temperature versus the outer radius of the liquid-filled annulus. Explain the relative insensitivity of the battery surface temperature to the size of the annulus for $15 \text{ mm} \leq r_o \leq 30$ mm.

9.79 Free convection occurs between concentric spheres. The inner sphere is of diameter $D_i = 50$ mm and temperature $T_i = 50^\circ\text{C}$, while the outer sphere is maintained at $T_o = 20^\circ\text{C}$. Air is in the gap between the spheres. What outer sphere diameter is required so that the convection heat transfer from the inner sphere is the same as if it were placed in a large, quiescent environment with air at $T_\infty = 20^\circ\text{C}$?

9.80 Consider the phase change material (PCM) of Problems 8.38 and 9.42. The PCM is housed in a long, horizontal, and insulated cylindrical enclosure of diameter $D_e = 200$ mm, which in turn includes a concentric, heated inner cylinder of diameter $D_i = 30$ mm. Initially, the PCM is entirely solid and at its phase change temperature. The inner cylinder temperature is suddenly raised to $T_h = 50^\circ\text{C}$. Assuming the PCM melts to form an expanding concentric liquid region about the heated tube such as the one shown in the schematic, determine how long it takes to melt half of the PCM.



9.81 The human eye contains aqueous humor, which separates the external cornea and the internal iris–lens structure. It is hypothesized that, in some individuals, small flakes of pigment are intermittently liberated from the iris and migrate to, and subsequently damage, the cornea. Approximating the geometry of the enclosure formed by the cornea and iris–lens structure as a pair of concentric hemispheres of outer radius $r_o = 10$ mm and inner radius $r_i = 7$ mm, respectively, investigate whether free convection can occur in the aqueous humor by evaluating the effective thermal conductivity ratio, k_{eff}/k . If free convection can occur, it is possible that the damaging particles are advected from the iris to the cornea. The iris–lens structure is at the core temperature, $T_i = 37^\circ\text{C}$, while the cornea temperature is measured to be $T_o = 34^\circ\text{C}$. The properties of the aqueous humor are $\rho = 990 \text{ kg/m}^3$, $k = 0.58 \text{ W/m} \cdot \text{K}$, $c_p = 4.2 \times 10^3 \text{ J/kg} \cdot \text{K}$, $\mu = 7.1 \times 10^{-4} \text{ N} \cdot \text{s/m}^2$, and $\beta = 3.2 \times 10^{-4} \text{ K}^{-1}$.

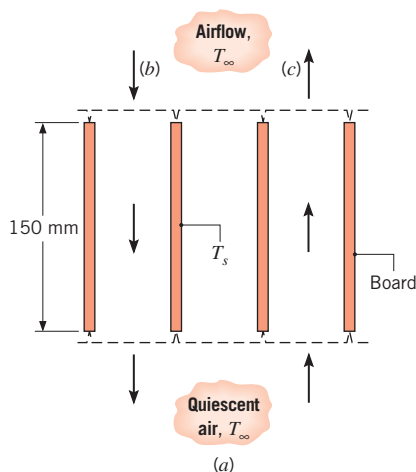


Mixed Convection

9.82 A horizontal, 25-mm diameter cylinder is maintained at a uniform surface temperature of 35°C . A fluid with a velocity of 0.05 m/s and temperature of 20°C is in cross flow over the cylinder. Determine whether heat transfer by free convection will be significant for (i) air, (ii) water, (iii) engine oil, and (iv) mercury.

9.83 According to experimental results for parallel airflow over a uniform temperature, heated vertical plate, the effect of free convection on the heat transfer convection coefficient will be 5% when $Gr_L/Re_L^2 = 0.08$. Consider a heated vertical plate 0.5 m long, maintained at a surface temperature of 55°C in atmospheric air at 30°C . What is the minimum vertical velocity required of the airflow such that free convection effects will be less than 5% of the heat transfer rate?

9.84 A vertical array of circuit boards of 150-mm height is to be air cooled such that the board temperature does not exceed 60°C when the ambient temperature is 25°C .



Assuming isothermal surface conditions, determine the allowable electrical power dissipation per board for the cooling arrangements:

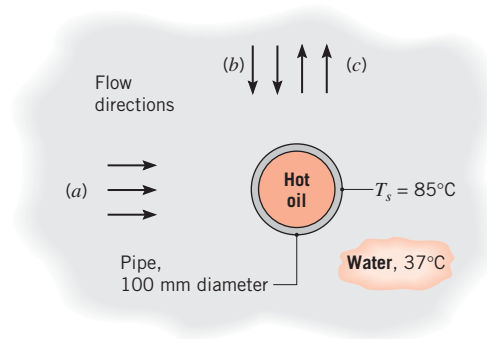
- Free convection only (no forced airflow).
- Airflow with a downward velocity of 0.6 m/s.
- Airflow with an upward velocity of 0.3 m/s.
- Airflow with a velocity (upward or downward) of 5 m/s.

9.85 A horizontal, 25-mm-diameter cylinder is maintained at a uniform surface temperature of 35°C . Air at atmospheric pressure with a velocity of 0.10 m/s and temperature 20°C is in cross flow over the cylinder. Calculate the convection heat rate per unit cylinder length. How might the cylinder be maintained at the warm temperature if its surface is adiabatic?

9.86 A probe, used to measure the velocity of air in a low-speed wind tunnel, is fabricated of an $L = 100$ mm long, $D = 8$ -mm outside diameter horizontal aluminum tube. Power resistors are inserted into the stationary tube and dissipate $P = 1.5$ W. The surface temperature of the tube is determined experimentally by measuring the emitted radiation from the exterior of the tube. To maximize surface emission, the exterior of the tube is painted with flat black paint having an emissivity of $\varepsilon = 0.95$.

- For air at a temperature and cross flow velocity of $T_\infty = 25^\circ\text{C}$, $V = 0.1$ m/s, respectively, determine the surface temperature of the tube. The surroundings temperature is $T_{\text{sur}} = 25^\circ\text{C}$.
- For the conditions of part (a), plot the tube surface temperature versus the cross flow velocity over the range $0.05 \text{ m/s} \leq V \leq 1 \text{ m/s}$.

9.87 A horizontal 100-mm-diameter pipe passing hot oil is to be used in the design of an industrial water heater. Based on a typical water draw rate, the velocity over the pipe is 0.5 m/s. The hot oil maintains the outer surface temperature at 85°C and the water temperature is 37°C .



Investigate the effect of flow direction on the heat rate (W/m) for (a) horizontal, (b) downward, and (c) upward flow.

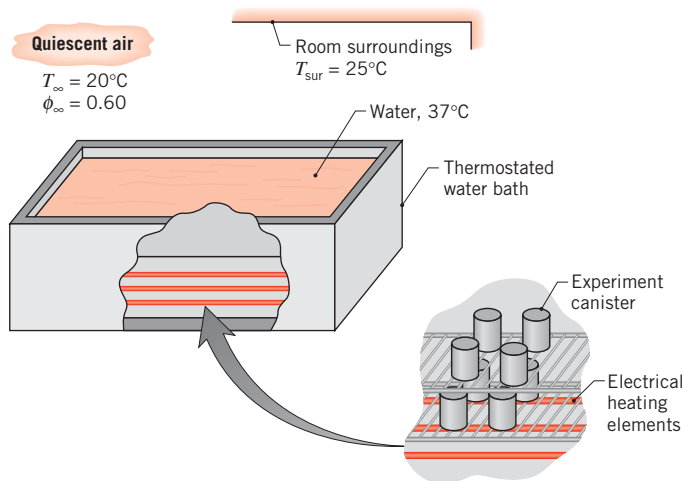
9.88 Determine the heat transfer rate from the steel plates of Problem 7.19 accounting for free convection from the plate surfaces. What is the corresponding rate of change of the plate temperature? Plot the heat transfer coefficient associated with free convection, forced convection, and mixed convection for air velocities ranging from $2 \leq u_\infty \leq 10$ m/s. The velocity of the plate is small compared to the air velocity.

9.89 An experiment involves heating a very small sphere that is suspended by a fine string in air with a laser beam in order to induce the highest sphere temperature possible. After inspecting Equation 9.64, a research assistant suggests inducing a uniform downward airflow to *exactly* offset free convection from the sphere, thereby minimizing heat losses and maximizing the steady-state sphere temperature. In the limiting case of a *very small* sphere, what is the minimum value of the convection heat transfer coefficient expressed in terms of the sphere diameter and thermal conductivity of the air?

Mass Transfer

9.90 A 205 mm $\times$ 245 mm sheet of wetted fabric is hung up to dry on a warm, sunny day. The still air is at a temperature of 30°C and relative humidity of 40%. To maximize the drying rate, should the sheet be hung with its long dimension or short dimension in the vertical direction? Determine the maximum drying rate when the sheet temperature is 26°C .

- 9.91** A water bath is used to maintain canisters containing experimental biological reactions at a uniform temperature of 37°C. The top of the bath has a width and length of 0.25 m and 0.50 m, respectively, and is uncovered to allow easy access for removal or insertion of the canisters. The bath is located in a draft-free laboratory with air at atmospheric pressure, a temperature of 20°C, and a relative humidity of 60%. The walls of the laboratory are at a uniform temperature of 25°C.



- (a) Estimate the rate of heat loss from the surface of the bath by radiation exchange with the surroundings.

- (b) Calculate the Grashof number using Equation 9.65, which can be applied to natural convection flows driven by temperature and concentration gradients. Use a characteristic length L that is appropriate for the exposed surface of the water bath.
- (c) Estimate the free convection heat transfer coefficient using the result for Gr_L obtained in part (b).
- (d) Invoke the heat and mass transfer analogy and use an appropriate correlation to estimate the mass transfer coefficient using Gr_L . Calculate the water evaporation rate on a daily basis and the rate of heat loss by evaporation.
- (e) Calculate the total rate of heat loss from the surface, and compare the relative contributions of the sensible, latent, and radiative effects. Review the assumptions made in your analysis, especially those relating to the heat and mass transfer analogy.

- 9.92** On a very still morning, the surface temperature of a lake used to cool the condenser of a power plant is 30°C while the air temperature is 23°C with a relative humidity of 80%. Assume a surroundings temperature of 290 K. The lake is nominally circular in shape with a diameter of approximately 5 km. Determine the rate of heat loss from the surface of the lake by radiation, free convection, and evaporation. This heat loss determines the capacity of the lake to cool the condenser. Justify why the heat transfer correlation you select is useful, even though Ra_L is outside of its specified range. *Hint:* See Problem 9.91.

